WRITE YOUR NAME:

MAC 2312 WRITTEN HOMEWORK #13

Due Thursday April 18th, in Canvas

Question 1. Evaluate the integral.

$$\int_0^{\pi/4} (1 + \tan x) \sec^2 x \, dx$$

Sub $u = \tan x \Rightarrow du = \sec^2 x \, dx$

If $x = 0$ then $u = \tan 0 = 0$

If $x = \frac{\pi}{4}$ then $u = \tan \frac{\pi}{4} = 1$

$$\int_{x=0}^{x=\pi/4} (1 + \tan x) \underbrace{\sec^2 x \, dx} = \int_{u=0}^{u=1} (1 + u) \, \underbrace{du}$$

$$= \left[u + \frac{u^2}{2} \right]_{u=0}^{u=1} = 1 + \frac{1}{2} = \frac{3}{2}$$

Question 2. Evaluate the integral.

$$\int_{\pi/6}^{\pi/2} \cos x \ln(\sin x)\, dx$$

METHOD 1. Start with substitution $u = \sin x \Rightarrow du = \cos x\, dx$

$x = \frac{\pi}{6} \Rightarrow u = \sin\frac{\pi}{6} = \frac{1}{2}$, $\quad x = \frac{\pi}{2} \Rightarrow u = \sin\frac{\pi}{2} = 1$

$$\int_{x=\pi/6}^{x=\pi/2} \ln(\sin x)\cdot \underbrace{\cos x\, dx}_{du} = \int_{u=1/2}^{u=1} \ln(u)\, du$$

Next, if you don't know the integral of ln, integrate by parts.

Rename: $\displaystyle\int_{t=1/2}^{t=1} \ln t\, dt \quad$ Try $\left.\begin{array}{l} u = \ln t \\ dv = 1\, dt \end{array}\right\} \Rightarrow \begin{array}{l} du = \frac{1}{t} dt \\ v = t \end{array}$

$$\int_{t=1/2}^{t=1} \underbrace{\ln t}_{u} \cdot \underbrace{dt}_{dv} = \Big[\underbrace{\ln t}_{u} \cdot \underbrace{t}_{v}\Big]_{t=1/2}^{t=1} - \int_{t=1/2}^{t=1} \underbrace{t}_{v} \cdot \underbrace{\frac{1}{t}\, dt}_{du}$$

$$= \Big[t \ln t\Big]_{t=1/2}^{t=1} - \int_{t=1/2}^{t=1} 1\, dt$$

$$= \underbrace{1 \ln 1}_{0} - \frac{1}{2}\ln\frac{1}{2} - \Big[t\Big]_{1/2}^{1} = -\frac{1}{2}\ln\frac{1}{2} - \frac{1}{2}$$

$$= \frac{1}{2}\ln 2 - \frac{1}{2} \quad \text{or} \quad \frac{\ln 2 - 1}{2}$$

Question 2. Evaluate the integral.

$$\int_{\pi/6}^{\pi/2} \cos x \ln(\sin x)\, dx$$

METHOD 2. Start with integration by parts.

$$\left.\begin{aligned} u &= \ln(\sin x) \\ dv &= \cos x\, dx \end{aligned}\right\} \Rightarrow \begin{aligned} du &= \frac{1}{\sin x} \cdot \cos x\, dx \\ v &= \sin x \end{aligned}$$

$$\int_{\pi/6}^{\pi/2} \underbrace{\ln(\sin x)}_{u} \cdot \underbrace{\cos x\, dx}_{dv} = \Big[\underbrace{\ln(\sin x)}_{u} \cdot \underbrace{\sin x}_{v}\Big]_{\pi/6}^{\pi/2} - \int_{\pi/6}^{\pi/2} \underbrace{\sin x}_{v} \cdot \underbrace{\frac{1}{\sin x} \cdot \cos x\, dx}_{du}$$

$$= \underbrace{\ln(\underbrace{\sin \tfrac{\pi}{2}}_{1})}_{0} \cdot \sin\frac{\pi}{2} - \ln(\underbrace{\sin\tfrac{\pi}{6}}_{\frac{1}{2}}) \cdot \underbrace{\sin\frac{\pi}{6}}_{\frac{1}{2}} - \int_{\pi/6}^{\pi/2} \cos x\, dx$$

$$= -\frac{1}{2} \underbrace{\ln\left(\frac{1}{2}\right)}_{= -\ln 2} - \Big[\sin x\Big]_{\pi/6}^{\pi/2}$$

$$= \frac{1}{2} \ln 2 - \Big(\underbrace{\sin\frac{\pi}{2}}_{1} - \underbrace{\sin\frac{\pi}{6}}_{\frac{1}{2}}\Big)$$

$$= \frac{1}{2}\ln 2 - \frac{1}{2} \quad \text{or} \quad \frac{\ln 2 - 1}{2}$$

Question 3. Evaluate the integral.

$$\int xe^{\sqrt{x}}\,dx$$

Substitute $t = \sqrt{x} = x^{1/2} \Rightarrow dt = \frac{1}{2}x^{-1/2}dx = \frac{1}{2\sqrt{x}}dx$

$$\text{Integral} = \int \underbrace{x}_{t^2}\,\underbrace{e^{\sqrt{x}}}_{e^t} \cdot \underbrace{2\sqrt{x}}_{2t} \cdot \underbrace{\frac{1}{2\sqrt{x}}\,dx}_{dt} = 2\int t^3 e^t\,dt$$

Then do integration by parts several times.

First: $u = t^3,\ dv = e^t dt \Rightarrow du = 3t^2 dt,\ v = e^t$

$$\text{Integral} = 2t^3e^t - 2\int e^t \cdot 3t^2 dt = 2t^3e^t - 6\int t^2e^t dt$$

Next: $u = t^2,\ dv = e^t dt \Rightarrow du = 2t\,dt,\ v = e^t$

$$\text{Integral} = 2t^3e^t - 6\left(t^2e^t - \int e^t \cdot 2t\,dt\right)$$

$$= 2t^3e^t - 6t^2e^t + 12\int te^t dt$$

Finally: $u = t,\ dv = e^t dt \Rightarrow du = dt,\ v = e^t$

$$\text{Integral} = 2t^3e^t - 6t^2e^t + 12\left(te^t - \underbrace{\int e^t dt}_{e^t}\right)$$

$$= 2t^3e^t - 6t^2e^t + 12te^t - 12e^t + C$$

or $(2t^3 - 6t^2 + 12t - 12)e^t + C$

$$= \left(2x^{3/2} - 6x + 12x^{1/2} - 12\right)e^{\sqrt{x}} + C$$