WRITE YOUR NAME:

MAC 2312 Quiz 6
Thursday February 1st

Let A be the region bounded by $y = x$ and $y = x^2$. Find the volume obtained by revolving A around the y-axis.

$y = f(x)$

Intersections? $x = x^2 \Rightarrow x^2 = x \Rightarrow x^2 - x = 0$

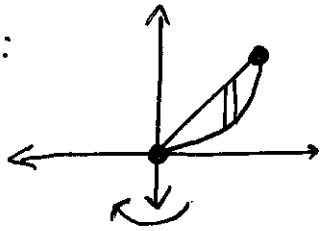
$\Rightarrow x(x-1) = 0 \Rightarrow x = 0, x = 1$

Top curve and bottom curve? Try test input $x = \frac{1}{2}$

Then $y = x \Rightarrow y = \frac{1}{2}$ ← bigger output

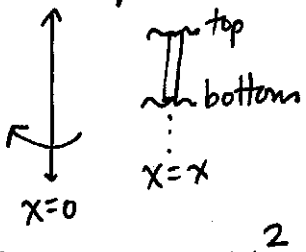
$y = x^2 \Rightarrow y = (\frac{1}{2})^2 = \frac{1}{4}$ ← smaller output

Picture:



Vertical slice around vertical axis

Slice parallel to axis → SHELLS



$h = (\overbrace{\text{top curve}}^{y=x}) - (\overbrace{\text{bottom curve}}^{y=x^2}) = x - x^2$

$r = (\underbrace{\text{typical slice}}_{x=x}) - (\underbrace{\text{axis of rev.}}_{x=0}) = x - 0 = x$

$$\text{Volume} = \int_{x=0}^{x=1} 2\pi r h \, dx = 2\pi \int_0^1 \underbrace{x}_{r} \underbrace{(x - x^2)}_{h} dx$$

$$= 2\pi \int_0^1 (x^2 - x^3) dx = 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1$$

$$= 2\pi \left(\frac{1}{3} - \frac{1}{4} \right) = 2\pi \left(\frac{4}{12} - \frac{3}{12} \right) = 2\pi \cdot \frac{1}{12} = \frac{\pi}{6}$$