WRITE YOUR NAME:

MAC 2312 Quiz 11
Thursday February 22nd

Evaluate the integral.

$$\int_0^{3/2} \frac{1}{\sqrt{9-x^2}}\,dx$$

Sub $x = 3\sin\theta$ ↓ $dx = 3\cos\theta\,d\theta$

If $x=0$ then $3\sin\theta=0 \Rightarrow \sin\theta=0 \Rightarrow \theta=0$

If $x=\frac{3}{2}$ then $3\sin\theta=\frac{3}{2} \Rightarrow \sin\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{6}$

$\frac{dx}{d\theta} = 3\cos\theta$

$$\int_{x=0}^{x=3/2} \frac{1}{\sqrt{9-x^2}}\,dx = \int_{\theta=0}^{\theta=\pi/6} \frac{1}{\sqrt{\underbrace{9-9\sin^2\theta}_{9(1-\sin^2\theta)}}} \cdot 3\cos\theta\,d\theta$$

$$= \int_{\theta=0}^{\theta=\pi/6} \frac{1}{\sqrt{9\cos^2\theta}}\,3\cos\theta\,d\theta = \int_{\theta=0}^{\theta=\pi/6} \frac{1}{3\cos\theta}\,3\cos\theta\,d\theta$$

$$= \int_{\theta=0}^{\theta=\pi/6} 1\,d\theta = \Big[\theta\Big]_{\theta=0}^{\theta=\pi/6} = \boxed{\frac{\pi}{6}}$$