

WRITE YOUR NAME:

MAC 2312 Quiz 12
Tuesday March 5th

Evaluate the integral using any correct method.

$$\int_4^5 \frac{2x-5}{x^2-5x+6} dx$$

METHOD 1. Sub $u = x^2 - 5x + 6 \Rightarrow du = (2x - 5)dx$

$$\frac{du}{dx} = 2x - 5$$

If $x = 4$ then $u = 4^2 - 5 \cdot 4 + 6 = 16 - 20 + 6 = 2$

If $x = 5$ then $u = 5^2 - 5 \cdot 5 + 6 = 25 - 25 + 6 = 6$

$$\int_{x=4}^{x=5} \frac{1}{\underbrace{x^2-5x+6}_u} \cdot \underbrace{(2x-5)dx}_{du} = \int_{u=2}^{u=6} \frac{1}{u} du$$

$$= \left[\ln|u| \right]_{u=2}^{u=6} = \ln|6| - \ln|2|$$

$$= \ln 6 - \ln 2 \quad \text{or} \quad \ln\left(\frac{6}{2}\right) = \ln 3$$

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METHOD 2. Partial fractions. $x^2-5x+6 = (x-2)(x-3)$

$$\text{Want } \frac{2x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$\downarrow$$
$$2x-5 = A(x-3) + B(x-2)$$

$$2x-5 = Ax - 3A + Bx - 2B$$

$$2x + (-5) = (Ax + Bx) + (-3A - 2B)$$

$$\underbrace{2x}_{m} + \underbrace{(-5)}_{eee} = \underbrace{(A+B)x}_{mmm} + \underbrace{(-3A-2B)}_{eeeeee}$$

$$\Rightarrow \begin{array}{l} \text{(i) } A+B = 2 \xrightarrow{\times 3} 3A+3B = 6 \\ \text{(ii) } -3A-2B = -5 \end{array} \quad \begin{array}{l} \downarrow \text{add} \\ \hline B = 1 \Rightarrow A = 1 \end{array}$$

$$\int_4^5 \frac{2x-5}{(x-2)(x-3)} dx = \int_4^5 \left(\frac{1}{x-2} + \frac{1}{x-3} \right) dx$$

$$= \left[\ln|x-2| + \ln|x-3| \right]_4^5$$

$$= (\ln|5-2| + \ln|5-3|) - (\ln|4-2| + \ln|4-3|)$$

$$= \ln 3 + \ln 2 - \ln 2 + \underbrace{\ln 1}_0 = \ln 3$$