

WRITE YOUR NAME:

MAC 2312 Quiz 13  
Thursday March 7th

Evaluate the integral.

$$\int_0^9 \frac{1}{\sqrt{9-x}} dx$$

Since  $f(x) = \frac{1}{\sqrt{9-x}}$  approaches infinity when  $x \rightarrow 9$ ,  
this is technically an IMPROPER INTEGRAL.

$$\text{Consider } \int_0^M \frac{1}{\sqrt{9-x}} dx = \int_0^M (9-x)^{-1/2} dx$$

$$\begin{aligned} \text{Sub } u &= 9-x \\ \Rightarrow du &= -1 dx \\ \Rightarrow -1 du &= dx \end{aligned}$$

$$\int_{x=0}^{x=M} (9-x)^{-1/2} dx = \int_{u=9}^{u=9-M} u^{-1/2} \cdot (-1) du$$

$$\begin{aligned} \text{If } x=0 &\text{ then } u=9 \\ \text{If } x=M &\text{ then } u=9-M \end{aligned}$$

$$\begin{aligned} &= \int_{9-M}^9 u^{-1/2} du = \left[ \frac{u^{1/2}}{1/2} \right]_{9-M}^9 = 2 \left[ \sqrt{u} \right]_{9-M}^9 = 2(\sqrt{9} - \sqrt{9-M}) \\ &= 2(3 - \sqrt{9-M}) \end{aligned}$$

$$\text{Therefore } \int_0^9 \frac{1}{\sqrt{9-x}} dx = \lim_{M \rightarrow 9} 2(3 - \sqrt{9-M})$$

$$= 2(3-0) = \boxed{6}$$

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Evaluate the integral.

$$\int_0^9 \frac{1}{\sqrt{9-x}} dx$$

If you don't treat this as an improper integral,  
you still get the right answer and I didn't penalize you.

$$\text{Sub } u = 9 - x \quad \text{If } x = 0 \text{ then } u = 9 - 0 = 9$$

$$du \downarrow = -1 dx \quad \text{If } x = 9 \text{ then } u = 9 - 9 = 0$$

$$-1 du = dx$$

$$\int_{x=0}^{x=9} \frac{1}{\sqrt{9-x}} dx = \int_{u=9}^{u=0} \frac{1}{\sqrt{u}} (-1) du = \int_0^9 \frac{1}{\sqrt{u}} du$$

$$= \int_0^9 u^{-1/2} du = \left[ \frac{u^{1/2}}{1/2} \right]_0^9 = 2 [\sqrt{u}]_0^9$$

$$= 2(\sqrt{9} - 0) = 2 \cdot 3 = \boxed{6}$$