WRITE YOUR NAME:

MAC 2312 Quiz 18
Thursday March 28th

Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$$ Series with positive terms

METHOD 1. Direct comparison. Notice $n^3+1 > n^3$.

Therefore $\frac{1}{n^3+1} < \frac{1}{n^3} \Rightarrow \sum \frac{1}{n^3+1} < \underbrace{\sum \frac{1}{n^3}}$

Known series: P-series, $p=3$
$p>1$ so series converges

By direct comparison, the given series CONVERGES

METHOD 2. Limit comparison. n^{th} term of given series is $a_n = \frac{1}{n^3+1}$

Try $b_n = \frac{1}{n^3}$. Then $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \left(\frac{1}{n^3+1} \div \frac{1}{n^3} \right)$

$= \lim_{n\to\infty} \left(\frac{1}{n^3+1} \cdot \frac{n^3}{1} \right) = \lim_{n\to\infty} \frac{n^3}{n^3+1} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n^3}} = \frac{1}{1+0} = 1$

Since $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$ is NOT 0 and NOT ∞,

can "jump"

We conclude $\sum a_n$ behaves the "same" as $\sum b_n$.

We know $\sum b_n = \sum \frac{1}{n^3}$ is a p-series with $p = 3 > 1$ which converges

By limit comparison, we conclude $\sum a_n = \sum \frac{1}{n^3+1}$ CONVERGES