

WRITE YOUR NAME:

MAC 2312 Quiz 21
Tuesday April 9th

Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n}$$

Try ratio test. $a_n = \frac{n^2}{3^n}$ $a_{n+1} = \frac{(n+1)^2}{3^{n+1}}$

$$\begin{aligned} R &= \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)^2}{3^{n+1}} \div \frac{n^2}{3^n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{(n+1)^2}{3^{n+1}} \cdot \frac{3^n}{n^2} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)^2}{n^2} \cdot \frac{3^n}{3^{n+1}} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{n^2 + 2n + 1}{n^2} \cdot \frac{1}{3} \right| = \lim_{n \rightarrow \infty} \left(\frac{n^2 + 2n + 1}{n^2} \cdot \frac{1}{3} \right) \\ &= \frac{1}{3} \lim_{n \rightarrow \infty} \frac{n^2 + 2n + 1}{n^2} = \frac{1}{3} \lim_{n \rightarrow \infty} \frac{n^2}{n^2} = \frac{1}{3} \cdot 1 = \frac{1}{3} \end{aligned}$$

Since $R = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{3}$ is LESS than 1,

we conclude by the ratio test that the given series CONVERGES.