

WRITE YOUR NAME:

MAC 2312 Quiz 23  
Thursday April 18th

Evaluate the integral using any correct method.

$$\int_0^1 x\sqrt{1-x^2} dx$$

METHOD 1.  $u$ -substitution.  $u = 1 - x^2 \Rightarrow du = -2x dx$   
 $\Rightarrow -\frac{1}{2} du = \underline{x dx}$ . If  $x=0$  then  $u = 1 - 0^2 = 1$   
If  $x=1$  then  $u = 1 - 1^2 = 0$

$$\int_{x=0}^{x=1} \sqrt{1-x^2} \cdot \underline{x dx} = \int_{u=1}^{u=0} \sqrt{u} \cdot \underline{-\frac{1}{2} du} = \frac{1}{2} \int_{u=0}^{u=1} u^{1/2} du$$
$$= \frac{1}{2} \left[ \frac{2}{3} u^{3/2} \right]_{u=0}^{u=1} = \frac{1}{3}$$

METHOD 2. Trig substitution.  $x = \sin \theta \Rightarrow dx = \cos \theta d\theta$   
If  $x=0$  then  $\sin \theta = 0 \Rightarrow \theta = 0$ , if  $x=1$  then  $\sin \theta = 1 \Rightarrow \theta = \pi/2$ .

$$\int_{x=0}^{x=1} x \sqrt{1-x^2} dx = \int_{\theta=0}^{\theta=\pi/2} \sin \theta \cdot \underbrace{\sqrt{1-\sin^2 \theta}}_{\cos^2 \theta} \cdot \underbrace{\cos \theta d\theta}_{\cos \theta d\theta}$$
$$= \int_{\theta=0}^{\theta=\pi/2} \sin \theta \cdot \underbrace{\cos \theta \cdot \cos \theta}_{\cos^2 \theta} d\theta. \text{ Next, sub } u = \cos \theta \Rightarrow du = -\sin \theta d\theta$$
$$\theta = 0 \Rightarrow u = \cos 0 = 1$$
$$\theta = \pi/2 \Rightarrow u = \cos \frac{\pi}{2} = 0$$

$$\int_{u=1}^{u=0} \frac{u^2 \cdot (-1) du}{\cos^2 \theta \sin \theta d\theta} = \int_{u=0}^{u=1} u^2 du = \left[ \frac{u^3}{3} \right]_{u=0}^{u=1} = \frac{1}{3}$$