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MAC 2312 Section U06 Test 1
Friday February 2nd
Total possible score: 20 points (2 points per page)

Question 1. Evaluate both sums.

$$\sum_{j=0}^{6}(-1)^j \qquad \sum_{m=2}^{4} 2^{m+1}$$

$$\sum_{j=0}^{6}(-1)^j = (-1)^0+(-1)^1+(-1)^2+(-1)^3+(-1)^4+(-1)^5+(-1)^6$$

$$= \underbrace{1 - 1}_{=0} + \underbrace{1 - 1}_{=0} + \underbrace{1 - 1}_{=0} + 1$$

$$= 1$$

$$\sum_{m=2}^{4} 2^{m+1} = 2^{2+1} + 2^{3+1} + 2^{4+1}$$

$$= 2^3 + 2^4 + 2^5$$

$$= 8 + 16 + 32$$

$$= 56$$

Question 2. Evaluate the integral. You may use geometry if it helps you.

$$\int_{-1}^{3} |2x-4| \, dx$$

Note $2x-4$ changes sign at $x=2$

$$\int_{-1}^{2} |2x-4| \, dx + \int_{2}^{3} |2x-4| \, dx$$

$$= \int_{-1}^{2} (4-2x) \, dx + \int_{2}^{3} (2x-4) \, dx$$

$$= \left[4x - x^2\right]_{-1}^{2} + \left[x^2 - 4x\right]_{2}^{3}$$

$$= \left[4x\right]_{-1}^{2} - \left[x^2\right]_{-1}^{2} + \left[x^2\right]_{2}^{3} - \left[4x\right]_{2}^{3}$$

$$= 4\left[x\right]_{-1}^{2} + \left[x^2\right]_{2}^{-1} + \left[x^2\right]_{2}^{3} + 4\left[x\right]_{3}^{2}$$

$$= 4(2-(-1)) + (1-4) + (9-4) + 4(2-3)$$

$$= 12 - 3 + 5 - 4$$

$$= 10$$

$$\frac{1}{x\sqrt{x}} = \frac{1}{x^1 \cdot x^{1/2}} = \frac{1}{x^{3/2}}$$

Question 3. Evaluate the integral.

$$\int_1^9 \frac{1}{x\sqrt{x}}\,dx$$

$$\int_1^9 x^{-3/2}\,dx$$

$$= \left[\frac{x^{-1/2}}{-1/2}\right]_1^9$$

$$= \left[-2x^{-1/2}\right]_1^9$$

$$= 2\left[x^{-1/2}\right]_9^1$$

$$= 2\left[\frac{1}{\sqrt{x}}\right]_9^1$$

$$= 2\left(\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{9}}\right) = 2\left(1 - \frac{1}{3}\right) = 2\cdot\frac{2}{3}$$

$$= \frac{4}{3}$$

θ	$\sin\theta$
0	0
$\pi/6$	$1/2$
$\pi/4$	$1/\sqrt{2}$
$\pi/3$	$\sqrt{3}/2$
$\pi/2$	1

Question 4. Evaluate the integral.

$$\int_{-\pi/4}^{\pi/4} \cos x \, dx$$

$$\Big[\sin x\Big]_{-\pi/4}^{\pi/4}$$

$$= \sin\frac{\pi}{4} - \sin\left(-\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}$$

Question 5. Evaluate the integral.

$$\int_1^2 \frac{2-x}{x}\,dx$$

$$\int_1^2 \left(\frac{2}{x} - \frac{x}{x}\right) dx$$

$$= \int_1^2 \left(2 \cdot \frac{1}{x} - 1\right) dx$$

$$= \Big[2\ln|x| - x\Big]_1^2$$

$$= \Big[2\ln|x|\Big]_1^2 - \Big[x\Big]_1^2$$

$$= 2\Big[\ln|x|\Big]_1^2 + \Big[x\Big]_2^1$$

$$= 2\Big(\underbrace{\ln|2|}_{=\ln 2} - \underbrace{\ln|1|}_{=\ln 1 = 0}\Big) + (1-2)$$

$$= 2\ln 2 - 1$$

Question 6. A particle moves with a velocity $v(t) = t^{2/3}$ (in meters per second). Its position function satisfies $s(8) = 0$. Find the position function $s(t)$.

$$s(t) = \int v(t)\,dt = \int t^{2/3}\,dt$$

$$= \frac{t^{5/3}}{5/3} + C = \frac{3}{5} t^{5/3} + C.$$

Next, find C. $s(8) = \frac{3}{5} \cdot 8^{5/3} + C$

$$= \frac{3}{5}(8^{1/3})^5 + C = \frac{3}{5} \cdot 2^5 + C$$

$$= \frac{3}{5} \cdot 32 + C = \frac{96}{5} + C.$$

But also $s(8) = 0$.

Therefore $\frac{96}{5} + C = 0$

$$C = -\frac{96}{5}$$

Position function is $s(t) = \frac{3}{5} t^{5/3} - \frac{96}{5}$.

Question 7. Find the average value of the function $f(x) = \sin x$ on the interval $[0, \pi]$.

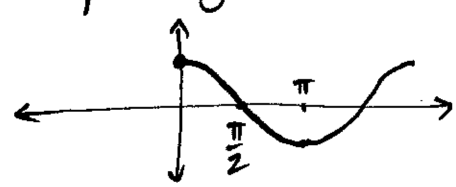
$$\frac{1}{\pi - 0}\int_0^{\pi} \sin x \, dx = \frac{1}{\pi}\int_0^{\pi} \sin x \, dx$$

$$= \frac{1}{\pi}\Big[-\cos x\Big]_0^{\pi}$$

$$= \frac{1}{\pi}\Big[\cos x\Big]_{\pi}^{0}$$

$$= \frac{1}{\pi}\left(\cos 0 - \cos \pi\right)$$

Graph of $y = \cos x$



$$= \frac{1}{\pi}\left(1 - (-1)\right)$$

$$= \frac{1}{\pi} \cdot 2 = \frac{2}{\pi}$$

Question 8. Evaluate the integral.

$$\int_1^7 \frac{x}{\sqrt{x^2+15}}\,dx$$

if $x=1$, then $u=1^2+15$
$=16$
if $x=7$, then $u=7^2+15$
$=49+15$
$=64$

Try substitution. $u = x^2+15$

$$\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$$
$$\frac{1}{2}du = x\,dx$$

$$\text{Integral} = \int_{x=1}^{x=7} \frac{1}{\sqrt{x^2+15}} \cdot x\,dx$$

$$= \int_{u=16}^{u=64} \frac{1}{\sqrt{u}} \cdot \frac{1}{2}\,du$$

$$= \frac{1}{2}\int_{u=16}^{u=64} u^{-1/2}\,du = \frac{1}{2}\left[\frac{u^{1/2}}{1/2}\right]_{u=16}^{u=64}$$

$$= \frac{1}{2}\left[2u^{1/2}\right]_{u=16}^{u=64} = \left[u^{1/2}\right]_{u=16}^{u=64} = 64^{1/2} - 16^{1/2}$$
$$= 8-4$$
$$= 4$$

Question 9. If $g(x)$ is defined by

$$g(x) = \int_1^{x^2} \frac{1}{\sqrt{t^3+1}}\, dt$$

find $g'(x)$.

$$g = \int_1^u \frac{1}{\sqrt{t^3+1}}\, dt \quad \text{and} \quad u = x^2$$

$$\frac{dg}{du} = \frac{1}{\sqrt{u^3+1}} \qquad \frac{du}{dx} = 2x$$

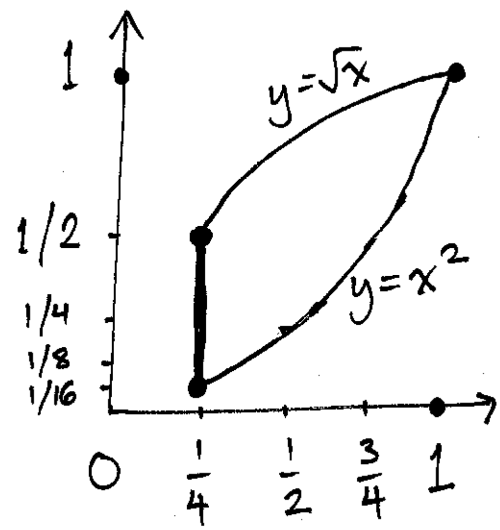
by Fundamental Theorem

$$g'(x) = \frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx} = \frac{1}{\sqrt{u^3+1}} \cdot 2x$$

$$= \frac{1}{\sqrt{(x^2)^3+1}} \cdot 2x = \frac{2x}{\sqrt{x^6+1}}$$

Question 10. Sketch the region bounded by the curves, and find its area.

$$y = x^2, \qquad y = \sqrt{x}, \qquad x = \frac{1}{4}, \qquad x = 1$$



$$\int_{1/4}^{1} \left(x^{1/2} - x^2\right) dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3} \right]_{1/4}^{1}$$

$$= \frac{2}{3}\left[x^{3/2}\right]_{1/4}^{1} + \frac{1}{3}\left[x^3\right]_{1}^{1/4}$$

$$= \frac{2}{3}\left(1 - \frac{1}{4^{3/2}}\right) + \frac{1}{3}\left(\frac{1}{4^3} - 1\right)$$

$$= \frac{2}{3}\left(1 - \frac{1}{8}\right) + \frac{1}{3}\left(\frac{1}{64} - 1\right)$$

$$= \frac{2}{3}\cdot\frac{7}{8} - \frac{1}{3}\cdot\frac{63}{64} = \frac{14}{24} - \frac{63}{192} = \frac{112}{192} - \frac{63}{192}$$

$$= 49/192$$