**Power series**

A power series is like a polynomial, but longer.

A power series centered at 0 looks like

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

A power series centered at $a$ looks like

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots$$

We can build power series step by step to approximate functions.

An expression of the form $c_0+c_1(x-a)$ can be a linear approximation of a particular function near $x = a$.

An expression of the form $c_0 + c_1(x-a) + c_2(x-a)^2$ can be a quadratic approximation of a particular function near $x = a$.

**Taylor polynomials**

A Taylor polynomial is a sum of powers of $(x-a)$ where the coefficient of $(x-a)^n$ is $f^{(n)}(a)/n!$

That is, the coefficient corresponding to the $n$th power is the $n$th derivative divided by $n$ factorial.

Why would we do this?

Suppose we want to build a polynomial or a power series that estimates $f(x) = e^x$. How might we go about it?

**Geometric series as power series**

Remember that the geometric series

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \cdots$$

converges to $\dfrac{a}{1-r}$ if $|r| < 1$.

It follows that the power series

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$$

converges to $\dfrac{1}{1-x}$ if $|x| < 1$.

**Convergence of power series**

A power series might converge for some values of $x$ and diverge for others.

We might want to know which $x$ values make a power series converge.

A power series centered at $a$ will definitely converge if $x = a$.

There will be an interval of $x$ values that make the series converge.

The number $a$ will be at the center of that interval of convergence.

We can use the *ratio test* to find out which values of $x$ make a power series converge.

**New power series from old**

If you have a power series, you might be able to obtain other power series by doing algebra, or by differentiating or integrating.

Can we find a power series for $\dfrac{\sin x}{x}$ or $e^{-x^2}$?

Can we then find a power series for $\displaystyle\int \frac{\sin x}{x}\,dx$ or $\displaystyle\int e^{-x^2}\,dx$?