EXAMPLES:

Find the Taylor series centered at 0 for the function $f(x) = \sqrt{x}$.

Find the Taylor series centered at 9 for the function $f(x) = \sqrt{x}$. Use this to estimate $\sqrt{9.1}$.

Find the Taylor series centered at 1 for the function $f(x) = \ln x$.

To find the interval of convergence of a Taylor series, we can use the ratio test.

EXAMPLE: Find the interval of convergence of the Taylor series for e^x:

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Recall that a geometric series can be written as a power series:

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x} \qquad \text{if } -1 < x < 1$$

By substituting and integrating, we can get power series for other functions!

Writing functions as Taylor series gives us alternative ways to evaluate cumbersome limits.

EXAMPLE: Evaluate the limits.

$$\lim_{x \to 0} \frac{2\cos 2x - 2 + 4x^2}{2x^4}$$

$$\lim_{x \to 0} \frac{\ln(1+x) - x + x^2/2}{x^3}$$

Taylor series give us alternative ways to approximate definite integrals. For example:

$$\int_0^2 e^{-x^2}\, dx = ?$$