WRITE YOUR NAME:

MAC 2313 B51 Spring 2024

Written homework #1

Due Tuesday January 16th, in Canvas

Question 1. Find the distance between the points $(-5, 4, 2)$ and $(-7, 2, 1)$.

$$\sqrt{(\underbrace{-5-(-7)}_{\substack{-5+7 \\ =2}})^2 + (4-2)^2 + (2-1)^2}$$

$$= \sqrt{2^2 + 2^2 + 1^2} = \sqrt{4+4+1} = \sqrt{9} = \boxed{3}$$

Question 2. Find a unit vector in the same direction as $\langle 2, 3, -6 \rangle$.

If $\vec{v} = \langle 2, 3, -6 \rangle$

then $|\vec{v}| = \sqrt{2^2 + 3^2 + (-6)^2}$

$= \sqrt{4 + 9 + 36} = \sqrt{49} = 7.$

So, a unit vector in the same direction as $\vec{v}$

is $\dfrac{1}{|\vec{v}|}\vec{v} = \dfrac{1}{7}\langle 2, 3, -6 \rangle = \left\langle \dfrac{2}{7}, \dfrac{3}{7}, \dfrac{-6}{7} \right\rangle.$

Question 3. Compute the dot product of the vectors $\mathbf{u} = \langle 4, 1, 1\rangle$ and $\mathbf{v} = \langle 1, -1, 0\rangle$, and also find the angle between the vectors.

$$\vec{u}\cdot\vec{v} = 4\cdot 1 + 1\cdot(-1) + 1\cdot 0$$
$$= 4 - 1 + 0 = 3$$

Fact: $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$ where θ = angle between $\vec{u}$ and $\vec{v}$

$$|\vec{u}| = \sqrt{4^2+1^2+1^2} = \sqrt{16+1+1} = \sqrt{18}$$

$$|\vec{v}| = \sqrt{1^2+(-1)^2+0^2} = \sqrt{1+1+0} = \sqrt{2}$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{v}|} = \frac{3}{\sqrt{18}\sqrt{2}} = \frac{3}{\sqrt{36}} = \frac{3}{6} = \frac{1}{2}$$

Therefore $\theta = \arccos\frac{1}{2} = \boxed{\frac{\pi}{3}}$

Question 4. Calculate the projection of the vector $\mathbf{u} = \langle -1, 4 \rangle$ onto the vector $\mathbf{v} = \langle -4, 2 \rangle$. Also draw a rough sketch to see whether your answer seems reasonable.

$$\text{proj}_{\vec{v}}\, \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v}$$

$$\vec{u} \cdot \vec{v} = (-1)(-4) + 4 \cdot 2 = 4 + 8 = 12$$

$$\vec{v} \cdot \vec{v} = (-4)^2 + 2^2 = 16 + 4 = 20$$

$$\text{So } \text{proj}_{\vec{v}}\, \vec{u} = \frac{12}{20} \vec{v} = \frac{3}{5} \vec{v} = \frac{3}{5} \langle -4, 2 \rangle$$

$$= \left\langle -\frac{12}{5}, \frac{6}{5} \right\rangle \quad \text{or} \quad \langle -2.4, 1.2 \rangle$$