WRITE YOUR NAME:

MAC 2313 B51 Spring 2024

Written homework #2

Due Tuesday January 23rd, in Canvas

Question 1. Evaluate the cross product of the vectors $\mathbf{u} = \langle 1, 2, 3 \rangle$ and $\mathbf{v} = \langle -1, 0, 2 \rangle$. Also evaluate $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$, and explain why it is equal to zero.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ -1 & 0 & 2 \end{vmatrix} = \langle 2\cdot 2 - 3\cdot 0, 3(-1) - 1\cdot 2, 1\cdot 0 - 2(-1) \rangle$$

$$= \langle 4-0, -3-2, 0+2 \rangle$$

$$= \langle 4, -5, 2 \rangle$$

Then $(\vec{u} \times \vec{v}) \cdot \vec{v} = \langle 4, -5, 2 \rangle \cdot \langle -1, 0, 2 \rangle$

$$= 4(-1) - 5\cdot 0 + 2\cdot 2 = -4 + 0 + 4 = 0$$

It makes sense that $(\vec{u} \times \vec{v}) \cdot \vec{v} = 0$ because we know $\vec{u} \times \vec{v}$ is always perpendicular to $\vec{v}$.

Question 2. Find the area of the parallelogram that has the vectors $\mathbf{u} = \langle 3, -1, 0 \rangle$ and $\mathbf{v} = \langle 0, 3, 2 \rangle$ as two of its adjacent sides.

Fact: The area of that parallelogram is $|\vec{u} \times \vec{v}|$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 0 \\ 0 & 3 & 2 \end{vmatrix} = \langle -2-0, 0-6, 9-0 \rangle = \langle -2, -6, 9 \rangle$$

$$|\vec{u} \times \vec{v}| = \sqrt{(-2)^2 + (-6)^2 + 9^2} = \sqrt{4 + 36 + 81}$$

$$= \sqrt{121} = 11$$

Question 3. Find both the vector equation and the parametric equations of the line through $(-3, 4, 2)$ that is perpendicular to both $\mathbf{u} = \langle 1, 1, -5 \rangle$ and $\mathbf{v} = \langle 0, 4, 0 \rangle$.

We can get a direction perpendicular to both $\vec{u}$ and $\vec{v}$ using the cross product:

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -5 \\ 0 & 4 & 0 \end{vmatrix} = \langle 0-(-20), 0-0, 4-0 \rangle$$

$$= \langle 20, 0, 4 \rangle$$

If we want, we can use any scalar multiple as our direction vector. So we could also use $\langle 5, 0, 1 \rangle$ if we want.

Vector equation of line:

$$\langle x, y, z \rangle = \langle -3, 4, 2 \rangle + t\langle 5, 0, 1 \rangle$$

Parametric equations of line:

$$x = -3 + 5t$$
$$y = 4$$
$$z = 2 + t$$

Question 4. Find the equation of the plane passing through the points $(0, 1, 0)$, $(2, 1, 4)$, and $(-2, 1, 0)$.

Let $P = (0,1,0)$ $\quad Q = (2,1,4)$ $\quad R = (-2,1,0)$

Two vectors parallel to the plane are $\overrightarrow{PQ} = Q - P = (2,0,4)$
and $\overrightarrow{PR} = R - P = (-2,1,0) - (0,1,0) = (-2,0,0)$.

A vector normal to the plane is $\overrightarrow{PQ} \times \overrightarrow{PR} = (2,0,4) \times (-2,0,0)$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 4 \\ -2 & 0 & 0 \end{vmatrix} = \langle 0-0, -8-0, 0-0 \rangle = \langle 0, -8, 0 \rangle$$

Any scalar multiple can also be used as a normal vector.
So we could use $\langle 0, 1, 0 \rangle$.

Equation of the plane will have the form $\underbrace{0x + 1y + 0z = d}_{\text{simplifies to } y = d}$.

Plug in any of the given points to find d.

ANSWER: The equation of the plane is $y = 1$.

(This makes sense, as $y = 1$ is a plane, and the three given points all have a y-coordinate of 1!)