WRITE YOUR NAME:

MAC 2313 B51 Spring 2024

Written homework #5

Due Tuesday February 13th, in Canvas

Question 1. Find the domain of the function $f(x, y, z) = \ln(16-x^2-y^2-z^2)$. (Geometrically, what does the domain look like as a subset of $\mathbb{R}^3$?)

Must have $16-x^2-y^2-z^2 > 0$

($\ln$ is defined only for positive inputs)

So $x^2+y^2+z^2 < 16$

Domain is interior of sphere of radius 4 centered at origin

Question 2. Prove that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+4y^2}$$

If (x,y) approaches $(0,0)$ along the line $y=0$ then

$$\frac{xy}{3x^2+4y^2} = \frac{x\cdot 0}{3x^2+4\cdot 0^2} = \frac{0}{3x^2} = 0$$

If (x,y) approaches $(0,0)$ along the line $y=x$ then

$$\frac{xy}{3x^2+4y^2} = \frac{x\cdot x}{3x^2+4x^2} = \frac{x^2}{7x^2} = \frac{1}{7}.$$

Since different paths to $(0,0)$ give different values, the limit does not exist.

Question 3. Find the domain of the function $f(x, y) = \dfrac{x}{\sqrt{x^2+y^2}}$.

Does the limit of $f(x, y)$ exist as (x, y) approaches $(0, 0)$?

We know x^2+y^2 is positive for all (x,y) except $(0,0)$.

The domain is all of $\mathbb{R}^2$ except the point $(0,0)$.

Suppose (x,y) approaches $(0,0)$ along the line $y=0$.

Then $$\frac{x}{\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+0^2}} = \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}$$

Note that $\frac{x}{|x|}$ actually approaches two different values:

$$\frac{x}{|x|} = 1 \text{ if } x>0, \text{ and } \frac{x}{|x|} = -1 \text{ if } x<0$$

Other paths give different values as well:

$$x=0 \Rightarrow \frac{x}{\sqrt{x^2+y^2}} = \frac{0}{\sqrt{0^2+y^2}} = \frac{0}{\sqrt{y^2}} = 0$$

$$y=x \Rightarrow \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+x^2}} = \frac{x}{\sqrt{2x^2}} = \frac{x}{\sqrt{2}\cdot|x|} = \frac{1}{\sqrt{2}} \text{ if } x>0$$

$$\left(-\frac{1}{\sqrt{2}} \text{ if } x<0\right)$$