WRITE YOUR NAME:

MAC 2313 B51 Spring 2024

Written homework #6

Due Tuesday February 20th, in Canvas

Question 1. Given $z = x \sin y$, $x = t^2$, and $y = 4t^3$, find dz/dt.

$z_x = 1 \cdot \sin y$
$= \sin y$

$z_y = x \cdot \cos y$

$x_t = 2t$

$y_t = 12t^2$

$\frac{dz}{dt} = z_x x_t + z_y y_t$

$= \sin y \cdot 2t + x \cos y \cdot 12t^2$

which can also be written

$\sin(4t^3) \cdot 2t + t^2 \cos(4t^3) \cdot 12t^2$

OR

$2t \sin(4t^3) + 12t^4 \cos(4t^3)$

OR

$2t\left(\sin(4t^3) + 6t^3 \cos(4t^3) \right)$

Question 2. Given $z = (x + 2y)^{10}$, $x = \sin^2 t$, and $y = (3t + 4)^5$, find dz/dt.

$$\frac{\partial z}{\partial x} = 10(x+2y)^9 \cdot 1 \qquad \frac{dx}{dt} = 2\sin t\cos t$$

$$\frac{\partial z}{\partial y} = 10(x+2y)^9 \cdot 2 = 20(x+2y)^9 \qquad \frac{dy}{dt} = 5(3t+4)^4 \cdot 3 = 15(3t+4)^4$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$= 10(x+2y)^9 \cdot 2\sin t\cos t + 20(x+2y)^9 \cdot 15(3t+4)^4$$

or $20(x+2y)^9 \sin t\cos t + 300(x+2y)^9(3t+4)^4$

or $20(x+2y)^9\left(\sin t\cos t + 15(3t+4)^4\right)$

Could replace x with $\sin^2 t$ and y with $(3t+4)^5$ but it's a lot to write

Question 3. Compute the gradient of the function $f(x, y) = 2 + 3x^2 - 5y^2$ and evaluate it at the point $(2, -1)$.

$$f_x = 0 + 6x - 0 = 6x$$

$$f_y = 0 + 0 - 10y = -10y$$

Gradient at general point is

$$\nabla f = \nabla f(x,y) = (f_x, f_y) = (6x, -10y)$$

Gradient at the point $(2,-1)$ is

$$\nabla f(2,-1) = (6 \cdot 2, -10 \cdot (-1))$$

$$= (12, 10)$$

Question 4. Compute the directional derivative of the function $f(x, y) = x^2 - y^2$ at the point $(-1, -3)$ in the direction of $\underbrace{\langle 3/5, -4/5 \rangle}$.

This is a unit vector

$$\left(\frac{3}{5}\right)^2 + \left(\frac{-4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1$$

$$f_x = 2x \qquad f_y = -2y$$

$$\nabla f(x, y) = (2x, -2y)$$

$$\nabla f(-1, -3) = (-2, 6)$$

The desired directional derivative is

$$\nabla f(-1, -3) \cdot \left(\frac{3}{5}, \frac{-4}{5}\right)$$

$$= (-2, 6) \cdot \left(\frac{3}{5}, \frac{-4}{5}\right)$$

$$= \frac{-6}{5} - \frac{24}{5} = \frac{-30}{5} = -6$$