**WRITE YOUR NAME:**

MAC 2313 B51 Spring 2024

Written homework #9

Due Tuesday March 19th, in Canvas

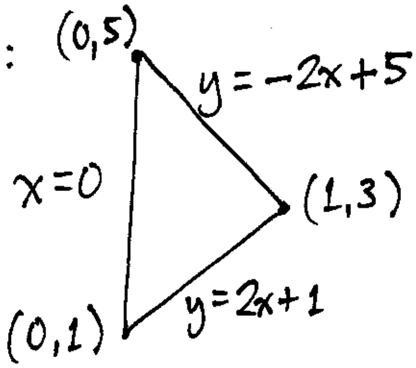
**Question 1.** Evaluate the integral

$$\iint_R xy\,dA$$

where $R$ is the region bounded by the lines $x = 0$, $y = 2x+1$, and $y = -2x+5$.

Intersections? $2x+1 = -2x+5 \Rightarrow 4x=4 \Rightarrow x=1$

Rough picture:




$$\int_{x=0}^{x=1}\int_{y=2x+1}^{y=-2x+5} xy\;dy\,dx = \int_0^1 \left[x\frac{y^2}{2}\right]_{y=2x+1}^{y=-2x+5} dx$$

$$= \int_0^1 \frac{x}{2}\left(\underbrace{(-2x+5)^2}_{4x^2-20x+25} - \underbrace{(2x+1)^2}_{4x^2+4x+1}\right)dx = \int_0^1 \frac{x}{2}(-24x+24)\,dx$$

$$= \int_0^1 (-12x^2+12x)\,dx = \left[-4x^3+6x^2\right]_0^1 = -4+6$$

$$= 2$$

**Question 2.** Evaluate the integral

$$\iint_R (x^2 + y^2)\, dA$$

where $R$ is the disk of radius 4 centered at the origin.

Much easier in polar coordinates. $0 \le r \le 4$, $0 \le \theta \le 2\pi$

$x^2 + y^2 = r^2$, $dA = r\,dr\,d\theta$

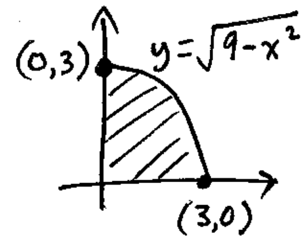
$$\int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=4} \underset{\substack{\uparrow \\ \text{from } x^2+y^2}}{r^2} \cdot \underbrace{r\,dr\,d\theta}_{\text{from } dA} = \int_0^{2\pi} \int_{r=0}^{r=4} r^3\,dr\,d\theta$$

$$= \int_0^{2\pi} \left[ \frac{r^4}{4} \right]_{r=0}^{r=4} d\theta = \int_0^{2\pi} 64\,d\theta = 64 \cdot 2\pi = 128\pi$$

**Question 3.** Evaluate the integral

$$\iint_R 2xy\,dA$$

where $R$ is the portion of the disk $x^2+y^2 \le 9$ lying in the first quadrant.




This one may be easier in rectangular.

METHOD 1 (rectangular) $\displaystyle\int_{x=0}^{x=3}\int_{y=0}^{y=\sqrt{9-x^2}} 2xy\,dy\,dx$

$$= \int_0^3 \left[xy^2\right]_{y=0}^{y=\sqrt{9-x^2}} dx = \int_0^3 x(9-x^2)\,dx = \int_0^3 (9x-x^3)\,dx$$

$$= \left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3 = \frac{9\cdot 3^2}{2} - \frac{3^4}{4} = \frac{81}{2} - \frac{81}{4} = \frac{81}{4}$$

METHOD 2 (polar) $0 \le r \le 3$, $0 \le \theta \le \pi/2$, $x = r\cos\theta$, $y = r\sin\theta$, $dA = r\,dr\,d\theta$

$$\int_{\theta=0}^{\theta=\pi/2}\int_{r=0}^{r=3} \underbrace{2\cdot r\cos\theta\cdot r\sin\theta}_{\text{from } 2xy}\cdot\underbrace{r\,dr\,d\theta}_{\text{from } dA} = \int_0^{\pi/2}\int_{r=0}^{r=3} 2r^3\sin\theta\cos\theta\,dr\,d\theta$$

$$= \int_0^{\pi/2}\left[\frac{2r^4}{4}\sin\theta\cos\theta\right]_{r=0}^{r=3} d\theta = \int_0^{\pi/2} \frac{81}{2}\sin\theta\cos\theta\,d\theta$$

Sub $u = \sin\theta \Rightarrow du = \cos\theta\,d\theta$

If $\theta = 0$ then $u = 0$, if $\theta = \frac{\pi}{2}$ then $u = 1$

$$= \int_{u=0}^{u=1} \frac{81}{2}\cdot \underset{\substack{\uparrow \\ \text{from } \sin\theta}}{u}\cdot \underset{\substack{\uparrow \\ \text{from } \cos\theta\,d\theta}}{du} = \left[\frac{81}{2}\cdot\frac{u^2}{2}\right]_{u=0}^{u=1} = \frac{81}{4}$$

**Question 4.** Evaluate the integral

$$\iint_R \frac{1}{1+x^2+y^2}\,dA$$

where $R$ is the disk of radius 2 centered at the origin.

Polar coordinates: $0 \le r \le 2$, $0 \le \theta \le 2\pi$, $x^2+y^2=r^2$, $dA = r\,dr\,d\theta$

$$\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2} \frac{1}{1+r^2}\, r\,dr\,d\theta = \int_{\theta=0}^{\theta=2\pi} \underbrace{\left(\int_{r=0}^{2} \frac{r\,dr}{1+r^2}\right)}_{\substack{\text{Sub } u=1+r^2 \\ \Rightarrow du = 2r\,dr \\ \frac{1}{2}du = r\,dr \\ r=0 \Rightarrow u=1 \\ r=2 \Rightarrow u=5}} d\theta$$

$$\int_{\theta=0}^{\theta=2\pi}\left(\int_{u=1}^{u=5} \frac{1}{2}\cdot\frac{1}{u}\,du\right)d\theta = \int_0^{2\pi}\left[\frac{1}{2}\ln u\right]_{u=1}^{u=5} d\theta$$

$$= \int_0^{2\pi} \frac{1}{2}\ln 5\,d\theta = 2\pi\cdot\frac{1}{2}\ln 5 = \pi \ln 5$$