WRITE YOUR NAME:

MAC 2313 B51 Spring 2024

Written homework #13

Due Tuesday April 16th, in Canvas

Question 1. Determine the length of the curve $\mathbf{r}(t) = \langle 2t, 3\sin 2t, 3\cos 2t \rangle$ on the interval $0 \le t \le 2\pi$.

$$\vec{r}\,'(t) = (2, 6\cos 2t, -6\sin 2t)$$

$$|\vec{r}\,'(t)| = \sqrt{2^2 + (6\cos 2t)^2 + (-6\sin 2t)^2}$$

$$= \sqrt{4 + \underbrace{36\cos^2 2t + 36\sin^2 2t}_{36(\underbrace{\cos^2 2t + \sin^2 2t}_{1})}} = \sqrt{4+36} = \sqrt{40}$$

$$\text{Length} = \int_{t=0}^{t=2\pi} ds = \int_{t=0}^{t=2\pi} |\vec{r}\,'(t)|\, dt$$

$$= \int_{t=0}^{t=2\pi} \sqrt{40}\, dt = \left[\sqrt{40}\cdot t\right]_{t=0}^{t=2\pi}$$

$$= \sqrt{40}\cdot 2\pi \quad \text{or} \quad 2\pi\sqrt{40} \quad \text{or} \quad 2\pi\cdot 2\sqrt{10} = 4\pi\sqrt{10}$$

Question 2. Find the linear approximation to $f(x,y) = 3 + \frac{x^2}{16} + \frac{y^2}{9}$ at the point $(x,y) = (-4,3)$, and use it to approximate $f(-3.999, 3.006)$.

$x_0 = -4$, $y_0 = 3$. $f_x = \frac{2x}{16} = \frac{x}{8}$, $f_y = \frac{2y}{9}$

$f_x(x_0,y_0) = \frac{-4}{8} = \frac{-1}{2}$, $f_y(x_0,y_0) = \frac{2\cdot 3}{9} = \frac{2}{3}$

$f(x_0,y_0) = 3 + \frac{(-4)^2}{16} + \frac{3^2}{9} = 3 + 1 + 1 = 5.$

Linear approximation is

$$f(x_0,y_0) + \underbrace{f_x(x_0,y_0)(x-x_0)}_{\text{change in } f \text{ caused by change in } x} + \underbrace{f_y(x_0,y_0)(y-y_0)}_{\text{change in } f \text{ caused by change in } y}$$

$$= 5 + \frac{-1}{2}(\underbrace{x-(-4)}_{x+4}) + \frac{2}{3}(y-3)$$

Therefore $f(-3.999, 3.006)$ is approximately

$5 + \frac{-1}{2}(-3.999+4) + \frac{2}{3}(3.006-3)$

$= 5 - \frac{1}{2}(0.001) + \frac{2}{3}(0.006)$

$= 5 - 0.0005 + 0.004 \quad = 5.0035$