

WRITE YOUR NAME:

MAC 2313 Quiz 5  
Tuesday January 30th

For the given curve, find an expression for the unit tangent vector at a general point.

$$\mathbf{r}(t) = \langle t^2, 2 \sin t, 2 \cos t \rangle$$

A tangent vector is  $\vec{r}'(t) = \langle 2t, 2\cos t, -2\sin t \rangle$ .

$$\text{Next, } |\vec{r}'(t)| = \sqrt{(2t)^2 + (2\cos t)^2 + (-2\sin t)^2}$$

$$= \sqrt{4t^2 + 4\cos^2 t + 4\sin^2 t} = \sqrt{4t^2 + 4}$$

$$\underbrace{4(\cos^2 t + \sin^2 t)}_{= 4 \cdot 1} = \sqrt{4(t^2 + 1)}$$

$$= 2\sqrt{t^2 + 1}$$

So, unit tangent vector is

$$\vec{T} = \frac{\vec{r}'}{|\vec{r}'|} = \frac{\langle 2t, 2\cos t, -2\sin t \rangle}{2\sqrt{t^2 + 1}}$$

$$= \left\langle \frac{t}{\sqrt{t^2 + 1}}, \frac{\cos t}{\sqrt{t^2 + 1}}, \frac{-\sin t}{\sqrt{t^2 + 1}} \right\rangle$$