WRITE YOUR NAME:

MAC 2313 Quiz 10
Tuesday February 20th

Find $\frac{dz}{dt}$ using any correct method.

$$z = x^2 y e^{xy^2}, \qquad x = t^7, \qquad y = t^{-3}$$

METHOD 1. $z = x^2 y \cdot e^{xy^2}$

$$z_x = \frac{\partial}{\partial x}(x^2 y)\cdot e^{xy^2} + x^2 y \cdot \frac{\partial}{\partial x}(e^{xy^2})$$

$$= 2xy e^{xy^2} + x^2 y e^{xy^2}\cdot y^2 = (2xy + x^2 y^3) e^{xy^2}$$

$$z_y = \frac{\partial}{\partial y}(x^2 y)\cdot e^{xy^2} + x^2 y \cdot \frac{\partial}{\partial y}(e^{xy^2})$$

$$= x^2 e^{xy^2} + x^2 y e^{xy^2}\cdot 2xy = (x^2 + 2x^3 y^2) e^{xy^2}$$

$$x_t = 7t^6, \quad y_t = -3t^{-4}$$

$$\frac{dz}{dt} = z_x x_t + z_y y_t$$

$$= (2xy + x^2 y^3) e^{xy^2}\cdot 7t^6 + (x^2 + 2x^3 y^2) e^{xy^2}\cdot(-3t^{-4})$$

WRITE YOUR NAME:

MAC 2313 Quiz 10
Tuesday February 20th

Find $\frac{dz}{dt}$ using any correct method.

$$z = x^2 y e^{xy^2}, \qquad x = t^7, \qquad y = t^{-3}$$

METHOD 2. Write z in terms of t first.

$$z = (t^7)^2 \cdot t^{-3} e^{t^7 (t^{-3})^2} = t^{14} t^{-3} e^{t^7 t^{-6}} = t^{11} e^t$$

$$\frac{dz}{dt} = \frac{d}{dt}(t^{11}) \cdot e^t + t^{11} \cdot \frac{d}{dt}(e^t)$$

$$= 11 t^{10} e^t + t^{11} e^t \quad \text{or } (11t^{10} + t^{11}) e^t \quad \text{or } t^{10} \cdot (11 + t) e^t$$

Compare with answer from Method 1:

$$(2xy + x^2 y^3) e^{xy^2} \cdot 7t^6 + (x^2 + 2x^3 y^2) e^{xy^2} \cdot (-3t^{-4})$$

$$= \left(2t^4 + \frac{t^{14} \cdot t^{-9}}{t^5}\right) e^t \cdot 7t^6 + \left(t^{14} + \frac{2t^{21} t^{-6}}{t^{15}}\right) e^t \cdot (-3t^{-4})$$

$$= (14t^{10} + 7t^{11}) e^t + (-3t^{10} - 6t^{11}) e^t$$

$$= (11t^{10} + t^{11}) e^t \quad \checkmark$$