WRITE YOUR NAME:

MAC 2313 Quiz 16
Thursday March 21st

Evaluate the integral by converting to polar coordinates.

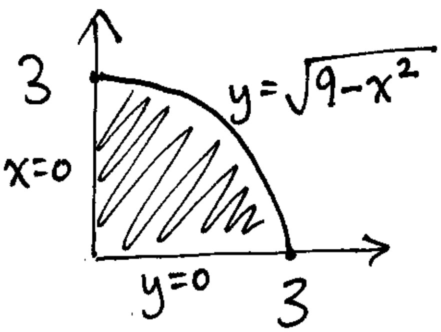
$$\int_0^3 \int_0^{\sqrt{9-x^2}} e^{x^2+y^2}\, dy\, dx$$

Region: $0 \le x \le 3$, $y=0$ is bottom curve, $y=\sqrt{9-x^2}$ is top curve

$$y^2 = 9 - x^2$$

$$x^2+y^2 = 9$$

part of CIRCLE
radius 3
center (0,0)



Same region in polar:
$0 \le r \le 3$, $0 \le \theta \le \pi/2$

$$x^2+y^2 = r^2$$

$$dA = r\,dr\,d\theta$$

$$\int_{\theta=0}^{\theta=\pi/2} \int_{r=0}^{r=3} \underbrace{e^{r^2}}_{e^{x^2+y^2}} \cdot \underbrace{r\,dr\,d\theta}_{dA} = \int_{\theta=0}^{\theta=\pi/2} \left(\int_{r=0}^{r=3} r e^{r^2}\, dr \right) d\theta$$

Long way: Sub $u=r^2$
$du = 2r\,dr$
$\frac{1}{2}du = r\,dr$

$$= \int_{\theta=0}^{\theta=\pi/2} \left[\underbrace{\frac{1}{2} e^{r^2}}_{} \right]_{r=0}^{r=3} d\theta = \int_{\theta=0}^{\theta=\pi/2} \underbrace{\left(\frac{1}{2}e^9 - \frac{1}{2} \right)}_{\text{CONSTANT}} d\theta$$

Correct parent, since its deriv. is $\frac{1}{2}e^{r^2} \cdot 2r$ by CHAIN RULE

$$= \int_{\theta=0}^{\theta=\pi/2} \frac{e^9-1}{2}\, d\theta = \frac{\pi}{2} \cdot \frac{e^9-1}{2} = \frac{\pi(e^9-1)}{4}$$