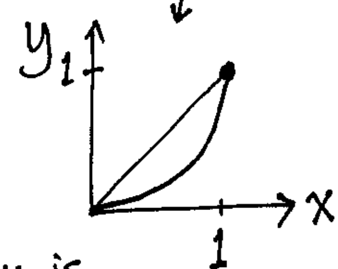
WRITE YOUR NAME:

MAC 2313 Quiz 17
Tuesday March 26th

Use a triple integral to find the volume enclosed by $z = 0$, $y = x^2$, $y = x$, and $z = x + y$.



y=x top
$y=x^2$ bottom

$z=0$ is "bottom surface", $z=x+y$ is "top surface"

$$\int_{x=0}^{x=1} \int_{y=x^2}^{y=x} \int_{z=0}^{z=x+y} 1 \, dz \, dy \, dx$$

$$= \int_{x=0}^{x=1} \int_{y=x^2}^{y=x} \Big[z \Big]_{z=0}^{z=x+y} dy \, dx = \int_{x=0}^{x=1} \int_{y=x^2}^{y=x} (x+y) \, dy \, dx$$

$$= \int_{x=0}^{x=1} \left[xy + \frac{y^2}{2} \right]_{y=x^2}^{y=x} dx = \int_{x=0}^{x=1} \left(x^2 + \frac{x^2}{2} - x^3 - \frac{x^4}{2} \right) dx$$

$$= \int_{x=0}^{x=1} \left(\frac{3x^2}{2} - x^3 - \frac{x^4}{2} \right) dx = \left[\frac{x^3}{2} - \frac{x^4}{4} - \frac{x^5}{10} \right]_{x=0}^{x=1}$$

$$= \frac{1}{2} - \frac{1}{4} - \frac{1}{10} = \frac{10}{20} - \frac{5}{20} - \frac{2}{20} = \frac{3}{20}$$