WRITE YOUR NAME:

MAC 2313 Quiz 18
Thursday March 28th

Evaluate the integral.

$$\int_0^{\pi/2}\int_0^{\pi/4}\int_0^{\cos\theta} \rho^2 \sin\theta \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{\pi/2}\int_0^{\pi/4} \left[\frac{\rho^3}{3}\sin\theta \right]_{\rho=0}^{\rho=\cos\theta} d\phi \, d\theta$$

$$= \int_0^{\pi/2}\int_0^{\pi/4} \frac{1}{3}\cos^3\theta \sin\theta \, d\phi \, d\theta$$

$$= \int_0^{\pi/2} \left[\frac{1}{3}\cos^3\theta \sin\theta \cdot \phi \right]_{\phi=0}^{\phi=\pi/4} d\theta$$

$$= \int_0^{\pi/2} \frac{1}{3}\cdot\frac{\pi}{4}\cos^3\theta \sin\theta \, d\theta = \frac{\pi}{12}\int_0^{\pi/2} \cos^3\theta \sin\theta \, d\theta$$

Sub $u = \cos\theta \Rightarrow du = -\sin\theta \, d\theta$ $\quad$ If $\theta = 0$ then $u = \cos 0 = 1$

$(-1)du = \sin\theta \, d\theta$ $\quad$ If $\theta = \frac{\pi}{2}$ then $u = \cos\frac{\pi}{2} = 0$

$$\frac{\pi}{12}\int_{u=1}^{u=0} u^3 \cdot (-1)\, du = \frac{\pi}{12}\int_0^1 u^3 \, du = \frac{\pi}{12}\left[\frac{u^4}{4} \right]_0^1$$

$$= \frac{\pi}{12}\cdot\frac{1}{4} = \frac{\pi}{48}$$