WRITE YOUR NAME:

MAC 2313 Quiz 19
Tuesday April 2nd

Find the volume of the region inside the cylinder $x^2+y^2=4$, above the plane $z=0$, and below the plane $y+z=3$.

Try cylindrical coordinates. $x^2+y^2 \le 4 \Rightarrow 0 \le r \le 2$, $0 \le \theta \le 2\pi$

Bottom surface $z=0$, top surface $z = 3-y = 3-r\sin\theta$

Remember $dV = r\,dz\,dr\,d\theta$ (Note the r)

$$\text{Volume} = \iiint 1\,dV = \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2}\int_{z=0}^{z=3-r\sin\theta} r\,dz\,dr\,d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2}\Big[rz\Big]_{z=0}^{z=3-r\sin\theta} dr\,d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2}\left(3r - r^2\sin\theta\right) dr\,d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi}\left[\frac{3r^2}{2} - \frac{r^3}{3}\sin\theta\right]_{r=0}^{r=2} d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi}\left(6 - \frac{8}{3}\sin\theta\right) d\theta = \left[6\theta + \frac{8}{3}\cos\theta\right]_{\theta=0}^{\theta=2\pi}$$

$$= 6\underbrace{\Big[\theta\Big]_0^{2\pi}}_{2\pi-0} + \frac{8}{3}\underbrace{\Big[\cos\theta\Big]_0^{2\pi}}_{\cos 2\pi - \cos 0 = 1-1=0} = 6\cdot 2\pi + \frac{8}{3}\cdot 0 = 12\pi$$