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MAC 2313 Quiz 20
Thursday April 4th

Evaluate the integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where \mathbf{F} is the vector field defined by

$$\mathbf{F}(x, y, z) = (8x^2yz, 5z, -4xy)$$

and C is the curve parametrized by

$$\mathbf{r}(t) = (t, t^2, t^3), \quad 0 \leq t \leq 1.$$

$$d\vec{\mathbf{r}} = \vec{\mathbf{r}}'(t) dt = (1, 2t, 3t^2) dt$$

$$\text{On } C, \text{ we have } \vec{\mathbf{F}} = (8t^2 \cdot t^2 \cdot t^3, 5t^3, -4t \cdot t^2) = (8t^7, 5t^3, -4t^3)$$

$$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{t=0}^{t=1} (8t^7, 5t^3, -4t^3) \cdot (1, 2t, 3t^2) dt$$

$$= \int_{t=0}^{t=1} (8t^7 + 10t^4 - 12t^5) dt$$

$$= \left[8 \frac{t^8}{8} + 10 \frac{t^5}{5} - 12 \frac{t^6}{6} \right]_{t=0}^{t=1} = \left[t^8 + 2t^5 - 2t^6 \right]_{t=0}^{t=1}$$

$$= 1 + 2 - 2 = 1$$