MAC 2313 Quiz 22
Tuesday April 16th

Let **F** be the vector field defined by

$$\mathbf{F}(x,y) = (2x+3y, 3x-2y)$$

and let C be the curve parametrized by

$$\mathbf{r}(t) = (x,y) = (\sin t, \cos t \sin^2 t), \qquad 0 \le t \le \pi/2.$$

Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ using any correct methods and/or shortcuts.

Could there be a potential function? Need $f(x,y)$ satisfying:

$f_x = 2x + 3y$ and $f_y = 3x - 2y$

$\downarrow$ $\downarrow$

$f = x^2 + 3xy + C_1$ $f = 3xy - y^2 + C_2$

can depend on y can depend on x

So $f = x^2 + 3xy - y^2$ works. By fundamental theorem of line integrals, we have $\int_C \vec{F} \cdot d\vec{r} = f(B) - f(A)$ where A = starting point of C, B = ending point of C.

$A = (\sin 0, \cos 0 \sin^2 0) = (0,0)$ $B = (\sin \frac{\pi}{2}, \cos\frac{\pi}{2} \sin^2 \frac{\pi}{2}) = (1,0)$

$$\int_C \vec{F} \cdot d\vec{r} = f(1,0) - f(0,0) = (1^2 + 3\cdot 1\cdot 0 - 0^2) - (0^2 + 3\cdot 0\cdot 0 - 0^2)$$

$$= 1$$

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Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ using any correct methods and/or shortcuts.

What happens if you try to integrate directly?

On C, we have $x = \sin t \Rightarrow \dfrac{dx}{dt} = \cos t$

$$y = \cos t \sin^2 t \Rightarrow \frac{dy}{dt} = (\cos t)' \sin^2 t + \cos t (\sin^2 t)'$$

$$= -\sin t \cdot \sin^2 t + \cos t \cdot 2 \sin t \cdot \cos t$$

$$= -\sin^3 t + 2\cos^2 t \sin t$$

Also on C, we have $\vec{F} = (2\underbrace{\sin t}_{x} + 3\underbrace{\cos t \sin^2 t}_{y}, 3\underbrace{\sin t}_{x} - 2\underbrace{\cos t \sin^2 t}_{y})$

$$\text{So} \int_C \vec{F} \cdot d\vec{r} = \int_0^{\pi/2} (2\sin t + 3\cos t \sin^2 t, 3\sin t - 2\cos t \sin^2 t) \cdot (\cos t, -\sin^3 t + 2\cos^2 t \sin t)\, dt$$

$$= \int_0^{\pi/2} \Big((2\sin t + 3\cos t \sin^2 t)(\cos t) + \underbrace{(3\sin t - 2\cos t \sin^2 t)(-\sin^3 t + 2\cos^2 t \sin t)}_{\text{Very cumbersome to expand manually}} \Big)\, dt$$