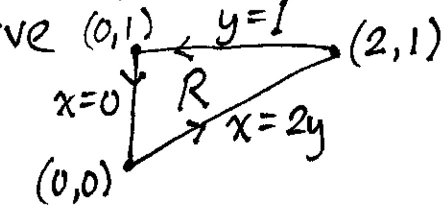
**WRITE YOUR NAME:**

MAC 2313 Quiz 23
Thursday April 18th

Calculate the flux of the vector field

$$\mathbf{F}(x, y) = (x^2y, 3x + y^2)$$ Components $M = x^2y$, $N = 3x + y^2$

across the curve $C$, where $C$ is the triangle bounded by the lines $x = 0$, $y = 1$, and $x = 2y$ (traversed counterclockwise). Closed curve




Most efficient method:
Use Divergence form of Green's Theorem.

$$M_x = 2xy,\ N_y = 2y.\ \text{Flux} = \iint_R (M_x + N_y)\, dA$$

$$= \int_{y=0}^{y=1} \int_{x=0}^{x=2y} (2xy + 2y)\, dx\, dy = \int_{y=0}^{y=1} \left[x^2y + 2xy\right]_{x=0}^{x=2y} dy$$

$$= \int_{y=0}^{y=1} (4y^3 + 4y^2)\, dy = \left[y^4 + \frac{4y^3}{3}\right]_0^1 = \frac{7}{3}$$

METHOD 2. Calculate flux "directly" as $\int_C M\,dy - N\,dx$

$C = C_1 \cup C_2 \cup C_3$

$C_1$: $(0,0)$ to $(2,1)$: $(x,y) = (2t, t)$, $0 \le t \le 1$, $dx = 2dt$, $dy = dt$

$C_2$: $(2,1)$ to $(0,1)$: $(x,y) = (2-2t, 1)$, $0 \le t \le 1$, $dx = -2dt$, $dy = 0$

$C_3$: $(0,1)$ to $(0,0)$: $(x,y) = (0, 1-t)$, $0 \le t \le 1$, $dx = 0$, $dy = -dt$

$$\text{Flux} = \int_{C_1} M\,dy - N\,dx + \int_{C_2} \cancel{M\,dy} - N\,dx + \int_{C_3} M\,dy - \cancel{N\,dx}$$

Then write everything in terms of $t$ and evaluate