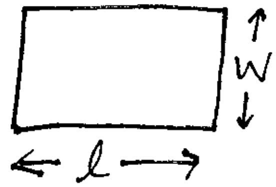
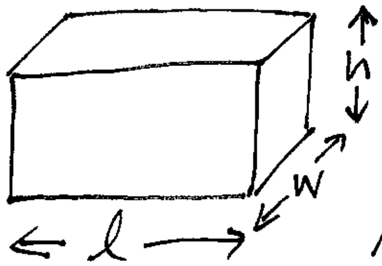
FUNDAMENTAL FACTS:



Area = $l \cdot w$



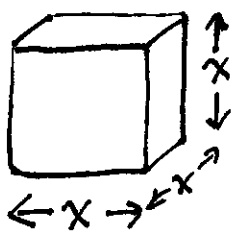
Volume = $l \cdot w \cdot h$

Also, don't <u>memorize</u> the formula for the surface area of a box. <u>Draw</u> the box, and <u>see</u> and <u>understand</u> what the surface area is.

Part 5

Related Rates

Question 5.1. A cube's surface area increases at the rate of 72 square inches per second. At what rate is the cube's volume changing when the edge length is $x = 3$ inches?



Surface area $= A = 6x^2$

GIVEN: $\dfrac{dA}{dt} = 72$

Volume $= V = x^3$ WANT $\dfrac{dV}{dt}$ when $x=3$.

$$A = 6x^2$$

$$\frac{d}{dt}(A) = \frac{d}{dt}(6x^2)$$

$$\underbrace{\frac{dA}{dt}}_{=72} = 12\underbrace{x}_{=3} \cdot \frac{dx}{dt}$$

$$72 = 36 \cdot \frac{dx}{dt}$$

$$\frac{dx}{dt} = 2$$

$$V = x^3$$

$$\frac{d}{dt}(V) = \frac{d}{dt}(x^3)$$

$$\underbrace{\frac{dV}{dt}}_{\text{WANT}} = 3\underbrace{x}_{x=3}{}^2 \cdot \underbrace{\frac{dx}{dt}}_{=2}$$

$$\frac{dV}{dt} = 3 \cdot 3^2 \cdot 2 = 54$$

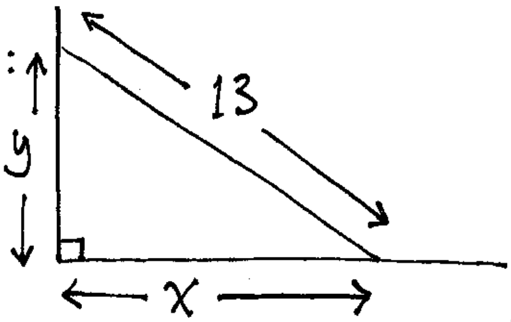
cubic inches per second

Question 5.2. A 13-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

(a) How fast is the top of the ladder sliding down the wall then?

(b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?

Picture:



GIVEN:

When $x = 12$, we have $\frac{dx}{dt} = 5$

(a) WANT $\frac{dy}{dt}$

We know area $= A = \frac{1}{2} \cdot x \cdot y$

(b) WANT $\frac{dA}{dt}$

We know $x^2 + y^2 = 13^2 = 169$

$$\frac{d}{dt}\left(x^2 + y^2\right) = \frac{d}{dt}(169)$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

$=12$ $\quad =5$ $\quad$?? $\quad$ WANT

If $x = 12$ then $x^2 + y^2 = 169$

$$12^2 + y^2 = 169$$

$$144 + y^2 = 169$$

$$y^2 = 25$$

$$y = 5$$

$$2 \cdot 12 \cdot 5 + 2 \cdot 5 \cdot \frac{dy}{dt} = 0$$

$$\Rightarrow \frac{dy}{dt} = -12$$

$\boxed{\text{(a) Top of ladder is sliding down at 12 ft per sec}}$

Then: $A = \frac{1}{2} \cdot x \cdot y$

$$\frac{d}{dt}(A) = \frac{d}{dt}\left(\frac{1}{2} \cdot x \cdot y\right)$$

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$$\underset{\text{WANT}}{\frac{dA}{dt}} = \frac{1}{2}\cdot\left(\underset{=5}{\frac{dx}{dt}}\cdot\underset{=5}{y} + \underset{=12}{x}\cdot\underset{=-12}{\frac{dy}{dt}}\right)$$

$$\frac{dA}{dt} = \frac{1}{2}\left(5\cdot 5 - 12\cdot 12\right)$$

$$= \frac{1}{2}\left(25 - 144\right) = \frac{1}{2}\cdot(-119)$$

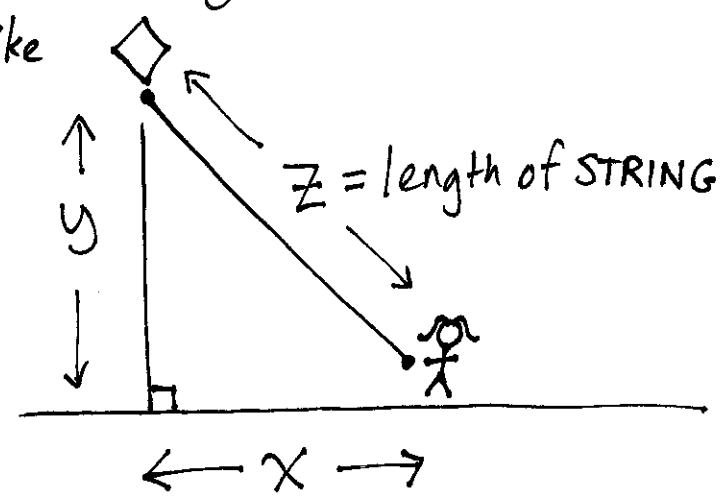
$$= -\frac{119}{2} \quad \text{or} \quad -59.5$$

$\boxed{\text{(b) Area is decreasing at 59.5 square feet per second}}$

Question 5.3. A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

First: Read, interpret, and think.
Probably kite is in the air.
Probably girl is on the ground.
Kite is allowed to move horizontally.

Picture should probably look like



GIVEN: $y = 300$ (height)

GIVEN: Kite moves horizontally away from girl, so distance x is increasing at a RATE of 25

$\Rightarrow$ given $\frac{dx}{dt} = 25$

WANT: rate at which she lets out the string, i.e. rate at which the distance z increases. WANT $\frac{dz}{dt}$ at the moment when $z = 500$.

Right-angled triangle $\Rightarrow$ we know $x^2 + y^2 = z^2$

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$$x^2 + y^2 = z^2$$

But y is constant in this problem

$$x^2 + 300^2 = z^2$$

$$\frac{d}{dt}\left(x^2 + 300^2\right) = \frac{d}{dt}\left(z^2\right)$$

$$\underset{?}{2x} \cdot \underset{25}{\frac{dx}{dt}} + 0 = \underset{500}{2z} \cdot \underset{\text{WANT}}{\frac{dz}{dt}}$$

If $z = 500$ then $x^2 + \underbrace{300^2}_{90000} = \underbrace{500^2}_{250000} \Rightarrow x = 400$ $\{x^2 = 160000\}$

(3-4-5 triangle)

$$2 \cdot 400 \cdot 25 = 2 \cdot 500 \cdot \frac{dz}{dt}$$

$$\frac{2 \cdot 400 \cdot 25}{2 \cdot 500} = \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{4}{5} \cdot 25 = 20$$

She must let out the string at 20 ft per sec (at that moment).