page 1

Calc I Prerequisite Review

Check your answers (can you justify them?) Did you understand the concepts!

→ e-mail me if you find a mistake

(1) a) $(3x-2)(3x+2)$ b) $(2x-\sqrt[3]{37})(4x^2+2\sqrt[3]{37}x+(\sqrt[3]{37})^2)$

c) $(x-1)(2x+1)(3x-2)$ d) $3(x+2)^2(2x+3)^3(4x+7)$

i) $9(2x-1)^{-1/3}(x+1)^3(2x+1)$ ← think about factoring out the terms with smallest exponent!

remark: We did not include complex numbers

Quad ≡ Quadrant (means)

Quad II, Quad I, III, IV, Quad II

(2) solve for all reals

a) answers in quadrant with positive sin (Quad I) $3x=\frac{\pi}{4}+2k\pi$ OR $3x=\pi-\frac{\pi}{4}+2k\pi$

$x=\frac{\pi}{12}+\frac{2}{3}k\pi$ or $x=\frac{\pi}{4}+\frac{2}{3}k\pi$ k is any integer $0,\pm1,\pm2,\dots$

b) cos is <0 in Quad II, Quad III $x=\frac{5\pi}{12}+k\pi,\ x=\frac{7\pi}{12}+k\pi$ $k=0,\pm1,\pm2,\dots$

cos<0, cos<0

c) $x=-\frac{\pi}{8}+\frac{k\pi}{2}$ another correct answer for c) $x=-\frac{\pi}{8}+k\pi,\ x=\frac{3\pi}{8}+k\pi$

d) domain of $\ln(2x+1)$ is $(-\frac{1}{2},\infty)$ solve and check it is in the domain

answer $x=\frac{e^{-2}-1}{2}$ it is $>-\frac{1}{2}$ because $e^{-2}>0$

e) Potential rational zeros are $\pm1,\pm2,\pm4,\pm8,\pm\frac{1}{3},\pm\frac{2}{3},\pm\frac{4}{3},\pm\frac{8}{3}$

check $-2,-\frac{2}{3}$ are zeros, we can factor $3x^4+14x^3+14x^2-8x-8$
$=(x+2)(3x+2)(x^2+2x-2)$

solve solutions $-2,-\frac{2}{3},-1\pm\sqrt{3}$

f) quadratic formula $x^2-4x-2=0$
$x=\frac{4\pm\sqrt{24}}{2}=2\pm\sqrt{6}$

g) domain = all reals take "ln" $\ln e^{3x-4}=\ln 3$
$3x-4=\ln 3$ solve $x=\frac{4+\ln 3}{3}$

(3) conjugate $\frac{(4-\sqrt{2x+1})(8+2\sqrt{3})}{64-12}=\frac{(4-\sqrt{2x+1})(4+\sqrt{3})}{26}$

(4) $\frac{25-(2x+1)}{2(x-14)(5+\sqrt{2x+1})}=\frac{12-x}{(x-14)(5+\sqrt{2x+1})}$ conjugate →

page 2 calc I (prerequisite review)

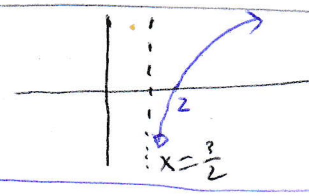
5) a) $f(3)=\frac{9-8}{18}=\frac{1}{18}$, $f(1)=\frac{3-8}{6}=\frac{-5}{6}$, $\frac{f(3)-f(1)}{3-1}=\frac{4}{9}$

b) $f(x)-f(1)=\frac{3x-8}{6x}+\frac{5}{6}=\frac{8(x-1)}{6x}$, $\frac{f(x)-f(1)}{x-1}=\frac{4}{3x}$

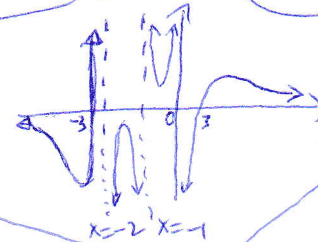
c) $f(1+h)-f(1)=\frac{3(1+h)-8}{6(1+h)}+\frac{5}{6}=\frac{8h}{6(1+h)}$, $\frac{f(1+h)-f(1)}{h}=\frac{4}{3(1+h)}$

---

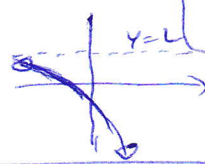
6) $y=f(x)$: asymptote $x=\frac{3}{2}$



$y=g(x)$: asymptotes V.A: $x=0$, $x=-2$, $x=-1$, H.A $y=0$

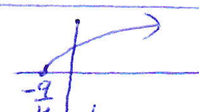


$y=2-3^{x+1}$ asymptote: H.A $y=2$

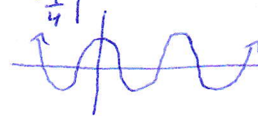


---

7) domain of $f=[-\frac{9}{4},\infty)$ Range $=[0,\infty)$



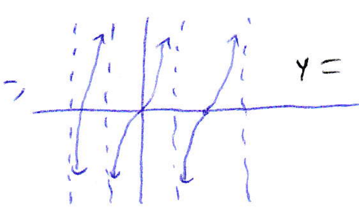
domain of $g=(-\infty,\infty)$ Range $=[-1,1]$



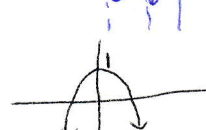
domain of $h=(-\infty,\infty)$ except odd multiples of $\frac{\pi}{2}$

$=\cdots(-\frac{3\pi}{2},-\frac{\pi}{2})\cup(-\frac{\pi}{2},\frac{\pi}{2})\cup(\frac{\pi}{2},\frac{3\pi}{2})\cdots$,

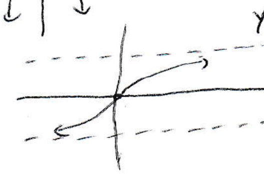
Range of $h=(-\infty,\infty)$



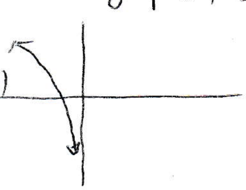
domain of $k=(-\infty,\infty)$, Range of $k=(-\infty,1]$



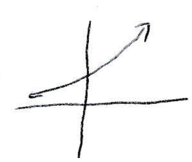
domain of $r(x)=(-\infty,\infty)$, Range of $r(x)=(-\frac{\pi}{2},\frac{\pi}{2})$



domain of $p(x)=(-\infty,0)$, Range of $p=(-\infty,\infty)$



domain of $t(x)=(-\infty,\infty)$ Range $=(0,\infty)$



page 3 Prerequisite calculus I

double angle formula

(8) $\cos(2\theta) = 2\cos^2\theta - 1 = 2(0.3)^2 - 1 = \frac{18}{100} - 1 = -\frac{82}{100} = -0.82$

Pythagorean Theorem $\rightarrow \sin^2(2\theta) = 1 - \cos^2(2\theta) = 1 - (0.82)^2$, $\sin(2\theta) = \pm\sqrt{1-(0.82)^2}$

---

(9) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\cos^2\theta + \sin^2\theta = 1$ — divide by $\cos^2\theta$, $1 + \tan^2\theta = \sec^2\theta$
— divide by $\sin^2\theta$, $\cot^2\theta + 1 = \csc^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$ from $\sin(a+b) = \sin a\cos b + \sin b\cos a$,

$\cos(a+b) = \cos a\cos b - \sin a\sin b$ — take $a=b$ $\cos 2a = \cos^2 a - \sin^2 a$

use $\cos^2 a = 1 - \sin^2\theta$

$\cos 2a = 1 - 2\sin^2 a$, $\cos 2a = 2\cos^2 a - 1$

$\tan\theta = \frac{1}{\cot\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

---

(10)

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$
0	0	1	0	1	D.N.E	D.N.E
$\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$	2	$\sqrt{3}$
$\pi/4$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	2	$\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
$\pi/2$	1	0	D.N.E	D.N.E	1	0

Use reference angle $0 \le \theta \le \frac{\pi}{2}$

$\cos(\pi+\theta) = -\cos\theta$

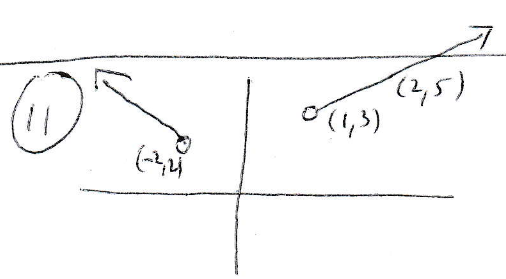
$\cos(\pi-\theta) = -\cos\theta$

$\sin(\pi+\theta) = -\sin\theta$

$\sin(\pi-\theta) = \sin\theta$

---

(11)



Domain $= (-\infty, -2) \cup (1, \infty)$ Range $= (2, \infty)$

page 4

Prerequisite calculus I

(12) $y=f(x)$, $y \le 0$ solve $x=-\sqrt{3-2y}$ to get $y=\frac{3-x^2}{2}$

$f^{-1}(x)=\frac{3-x^2}{2}$, domain $=(-\infty,0]$, Range $=(-\infty,\frac{3}{2}]$

(13) Review: Range of $\cos^{-1}(x)=[0,\pi]$, Range of $\sin^{-1}(x)=[-\frac{\pi}{2},\frac{\pi}{2}]$

Range of $\tan^{-1}x=(-\frac{\pi}{2},\frac{\pi}{2})$, Range of $\sec^{-1}x=[0,\pi]-\{\frac{\pi}{2}\}$

$\cos(\arcsin x)=?$ Pythagorean $\cos^2(\arcsin x)=1-(\sin(\arcsin x))^2=1-x^2$

since $-\frac{\pi}{2} \le \arcsin x \le \frac{\pi}{2}$, $\cos(\arcsin x)=+\sqrt{1-x^2}$

$\sin(\arccos x)=?$ Pythagorean $\sin(\arccos x)=+\sqrt{1-\cos^2 \arccos x}=\sqrt{1-x^2}$

$\tan(\arcsin x)=?=\frac{\sin(\arcsin x)}{\cos(\arcsin x)}=\frac{x}{\sqrt{1-x^2}}$

$\sin(\operatorname{arcsec} x)=\sqrt{1-\cos^2(\operatorname{arcsec} x)}$

$\sin(\operatorname{arcsec} x)=\sqrt{1-\frac{1}{x^2}}$

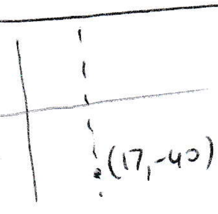
(14) slope $=m=\frac{7-3}{2-(-5)}=\frac{4}{7}$ Equation $y-7=\frac{4}{7}(x-2)$

(15) slope of $L_1: 3x+2y=-12$

$y=-\frac{3}{2}x-6$ → slope $m_1=-\frac{3}{2}$, $m_2 \perp m_1$ $m_2=-\frac{1}{m_1}=\frac{2}{3}$

Equation $y+6=\frac{2}{3}(x+1)$

(16)



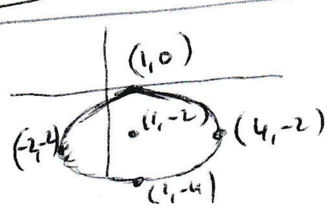
$x=17$

(17) slope of $-5x+8y=-40$ solve $y=\frac{5}{8}x-5$ slope $m_1=\frac{5}{8}$

Equation $y=\frac{5}{8}(x-8)+13$

(18) a) Ellipse centered at $(1,-2)$

$\frac{(x-1)^2}{9}+\frac{(y+2)^2}{4}=1$



b) parabola opens right

Vertex $=(2,-3)$