Under the Auspices of His Excellency Zine El Abidine Ben Ali President of the Republic of Tunisia



## من قرطاج إلى العالم مسـائِل الملكة عليـسة "دِيـدُونْ" المُتسـاوية الإحـاطة و تَشَعُّبـاتها الِرّياضية

مؤتمر عالمي حول الإرث الرياضي الهندسي للملكة عليسة مؤسسة قرطاج



Queen Dido Conference Carthage, Tunisia May 24-29, 2010 http://math.arizona.edu/~dido/



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#### INTERNATIONAL CONFERENCE ON THE ISOPERIMETRIC PROBLEM OF QUEEN DIDO AND ITS MATHEMATICAL RAMIFICATIONS

### WLM MY PP+EAN+ >L EOLM

We would like to welcome the guests of Tunisia who traveled from 20 countries and 64 universities and research centers in celebration of Queen Elissa-Dido and her isoperimetric legacy. This conference would not have seen the light if not for the international effort and the involvement of mathematicians from Tunisia, France, England, and North and South America. We especially celebrate the presence of many young mathematicians. We would like to recognize the sponsorship, without which this event would not take place, by the Tunisian Secretariat of State for Higher Education and Scientific Research, the U.S. National Science Foundation (NSF) and Office of International Science and Engineering (OISE), the Centre International de Mathématiques Pures et Appliquées (CIMPA), the CNRS-backed laboratoire international associé franco-maghrebin de mathématiques, l'Association pour la Promotion Scientifique de l'Afrique (APSA), Tunisian universities, various ministries (Higher Education and Scientific Research, Tourism, Culture, Transport, Women's Affairs) and the Tunis Science City.

The Tunisian scientific community has wholeheartedly embraced this international project which started as a dream. This event has been recognized by various mathematical bodies as one of their own: The American Mathematical Society (AMS), the European Mathematical Society (EMS), and the Tunisian Mathematical Society (SMT).



TUNIS, CARTHAGE AND SURROUNDINGS, 31 AUGUST 1535

Beyond the mathematical community, several Tunisian artists, historians and literary figures have embraced the conference since it strikes a core chord relating the foundational story of what ancient Arab historians called an "eternal" city, their eternally new city, "Qart Hadast" ( $\P P + F \Delta W +$ ). Dido has seen several metamorphoses beyond the "wandering" years escaping a far away Tyre (Sidon), beyond the ruse of the oxhide, beyond the legacies left by the Latin writers Virgil, Ovidius and others, and far beyond the "lamentations" of St. Augustine and "eternal perditions" of Tertullian and St. Cyprian, all natives of her city. She has outlived the many destructions of Carthage and has re-emerged like that eternal birth of fire, the phoenix (of which she is a true incarnation) a woman of science. The city she built is now a thriving vibrant country. With much vigor she is celebrated by Geometers, Analysts, Mathematical Physicists, and Engineers. Aspects of her problem and related ramifications bonding these fields will be explored during this conference. It is hoped that by holding the Queen Dido conference "be-arzat" ( $P > P \dashv F$ , Punic for "in situ"), many other legacies beyond the science, related to dialogue of civilizations and that which bonds us together, will be equally explored.

#### For the Conference Organizers: Najoua Gamara and Lotfi Hermi



THE PROBLEM OF QUEEN DIDO

A Historical Perspective



### 

The texts at the disposal of historians are still those of Timaeus and Justin. The latter seems to have reproduced the account of Pompeius Trogus. It is to this historian of Gaulic origins and Latin tongue that we owe the most detailed tradition. Justin is likely to have exploited, between the second and third century of the Christian era, the text of Pompeius Trogus who is thought to have had access to the writings of Timaeus of Tauromenium.

The myth is rooted in a moving account, of very high literary tenor, produced in a period when historical narrative was striving to adopt the form and style of literature. We have at hand bits of information relative to the circumstances which preceded the birth of Carthage: Pygmalion's ascension to power and his becoming king of Tyr, the assassination of Acerbas, high priest of Hercules/Melqart, the escape of Dido-Elissa, facilitated by Tyrian senators, the brief recess in Cyprus, the rapt of the virgins, the consultation with the priest of Jupiter or Juno (depending on the source), the ox-hide ruse, the encounter with a delegation from the more ancient settlement of Utica, choosing the location of the new city, the passion of King Jarbas for the Tyrian princess, and finally the death of the heroine foundress of Carthage.

Professor M'hamed Hassine Fantar Ben Ali's Chair for the Dialogue of Civilizations and Religions

#### A Mathematical Perspective

#### Overview of the Subject of Isoperimetry

by Mark Ashbaugh & Rafael Benguria $^{\rm 1}$ 



<sup>+</sup> he subject of isoperimetry has a long and eventful history, both for its impact on people's imaginations and society in general and for the impetus it has given to the study of various mathematical subjects.

Isoperimetry began with the problem confronted by Queen Dido, which was to find the shape of the boundary that should be laid down (using strips of oxhide) to enclose maximum area. If one assumes a straight coastline, then the answer, which was by all appearances discovered by Queen Dido, is to lay down the hide in the shape of a semi-circle.

One finds the problem of Queen Dido colorfully described, including various embellishments of the basic problem, in the expository account that Lord Kelvin gave in 1893 (see http://math.arizona.edu/~dido/lord-kelvin1894.html). If one takes account that land may vary in value, or that the coastline may be irregular, one can arrive at various more complicated problems. In a much more recent exposition, Hildebrandt and Tromba, in their book *The Parsimonious Universe: Shape and Form in the Natural World* (originally published as *Mathematics and Optimal Form*), give a much more detailed account of isoperimetric problems and their recurrence throughout history. In particular, it is interesting to see how many walled cities in the Middle Ages were constructed to have a nearly circular perimeter, or to see in general that the growth of many cities gave them a nearly circular form.

On the mathematical side, we find already in Euclid (around 300 BC) the proof that among rectangles of a given perimeter the one having the greatest area is the square. Also, various writers from antiquity speculated on optimal properties of the honeycombs of bees. When Thomas Hales proved in 2001 that regular hexagons provide a least-perimeter way to partition the plane into unit areas, it was the longest standing open problem in mathematics. As for 3D, Lord Kelvin proposed a solution consisting of relaxed, 14-sided, truncated octahedra. In 1994 D. Weaire and R. Phelan disproved Kelvin's conjecture by providing a new candidate using both 12- and 14-sided shapes.

Among the ancient Greeks who worked on the isoperimetric problem we mention Zenodorus (c. 200 - c. 140 BC) who wrote a now-lost treatise On Isoperimetric Figures and Ptolemy (c. 90 - c. 168 AD). It is thanks to Theon of Alexandria (c. 335 - c. 405 AD) who wrote a commentary on the work of Ptolemy that we know the results of Zenodorus. Al-Kindi, an Arab mathematician and the son of one of their kings, wrote in the 9th century A Treatise on Isoperimetric Figures and Isepiphanies, that is solids of given surface. There is also a lost treatise by al-Hasan ibn al-Haytham (965 - c. 1039). Abu Ja'far al-Khazin, commenting on Ptolemy's Almagest in the 10th century, generalized earlier works. Johannes de Sacrobosco (John of Holywood, c. 1195 - c. 1256 AD), an English scholar and astronomer, wrote Tractatus de Sphaera. A commentary on this treatise, dealing specifically with isoperimetry, can be found in Two New Sciences of Galileo Galilei published in 1638.

The mathematical study of the isoperimetric problem and related problems really began to take off with the advent of calculus, when people like Newton, Leibniz, the Bernoullis, and others developed systematic ways of attacking optimization problems based on the calculus, and within a few short years were attacking problems in the calculus of variations (that is, the problem of finding an optimizing path or shape of curve

 $<sup>^1{\</sup>rm with}$  contributions from Lennie Friedlander, Evans Harrell, Lotfi Hermi, and Frank Morgan.

from among some class of curves). For example, the brachistochrone problem was formulated by Johann Bernoulli and solved by Newton and both Bernoulli brothers, Jakob (James) and Johann (John). In the same period, the problem of the shape of a hanging chain (the catenary) was posed and solved, and Newton considered the shape of projectile which would give the least air resistance (the question of designing the optimal shape for the nose-cone of a rocket or missile), but without reaching definitive conclusions. Others, including US President Thomas Jefferson, considered questions such as the optimal shape for ploughshares.

In the century following the early development of calculus by Newton, Leibniz, the Bernoulli brothers, and others, the calculus of variations was brought to a relatively advanced state, especially from the point of view of direct solutions of problems, by Euler and Lagrange. The explicit solution of the classical isoperimetric problem could be derived in those terms (using variational theory with a constraint), and many other problems could be formulated and solved. Euler and Lagrange had shown that all of mechanics could be put into this framework, and that various physical and mathematical problems could be understood from the point of view of various optimization or variational principles (recall Fermat's principle of least time, or, more generally, the d'Alembert/Maupertuis principle of least action, for which Euler gave the definitive formulation). Almost a century later, Jacobi and Hamilton also made important contributions to this area, especially as regards mechanics.

In the nineteenth century Jakob Steiner attacked the classical isoperimetric problem using direct geometrical tools, which were very suggestive and instructive and led to many further developments. Around this time, however, Weierstrass realized that there could be subtle problems involved with attacking certain extremization problems, since it might be that no extremizer exists. Since that time it has been recognized that the existence question is where one must begin in attacking many problems from geometry and the calculus of variations. This led to various existence and uniqueness results, and to the so-called direct methods of the calculus of variations, wherein one tries to prove existence directly using extremizing sequences and various mathematical tools (developed by Weierstrass, Schwarz, Poincaré, Hilbert, and their contemporaries, and also more modern contributors, up to the present time).

A very useful development that came around the turn of the 20th century was Hurwitz's realization that the classical isoperimetric problem could be solved relatively simply in terms of Fourier series and some of their basic properties (e.g., Wirtinger's inequality).

The Fourier analysis approach to the isoperimetric inequality gave rise to further studies in higher dimensions where spherical harmonics take the place of Fourier series. This field is nicely summarized from a modern perspective in Groemer's book, *Geometric Applications of Fourier Series and Spherical Harmonics*.

Also in the nineteenth century the Belgian physicist J. Plateau experimented with soap films and conjectured that any wire loop (nice closed curve) bounds a soap film or minimal surface (of mean curvature 0). In 1936 J. Douglas won an inaugural Fields Medal for proving that every such loop bounds an immersed minimal disc, though his solution admitted self-intersections of a type which never occur in real soap films. Only with the advent of geometric measure theory with work of L. C. Young, E. De Giorgi, E. R. Reifenberg, H. Federer, W. Fleming, F. Almgren, J. Taylor, R. Hardt, L. Simon and others was the general existence of certain soap films established. It remains an open question today whether a smooth Jordan curve bounds a least-area soap film (" $(M, 0, \delta)$ -minimal set").

With a round soap bubble proved by Schwarz in 1884 to be the least-perimeter way to enclose a given volume of air, the next question was whether the double bubble that forms when two soap bubbles come together is the least-perimeter way to enclose and separate two given volumes of air. Years of progress by many mathematicians and undergraduates culminated in the 2002 proof by M. Hutchings, F. Morgan, M. Ritoré, and A. Ros.

From the point of view of engineering and design, perhaps the first truly interesting isoperimetric problem was to consider "the shape of the strongest column," a problem formulated by Lagrange in 1773 (but not fully solved until much later). In the mid 1800's T. Clausen was able to make his way around some of the points that Lagrange had stumbled over, though some questions have remained about the problem and its resolution up to recent times. See Steve Cox's Mathematical Intelligencer article, "The shape of the ideal column" to get a sense of where things stand currently. Several of the most pertinent recent contributors include J. Keller, I. Tadjbakhsh, M. Overton, and S. Cox. This problem has to do with the buckling of columns, and similar problems can be considered for horizontal beams under a variety of loads, and for plates and other structural members having greater geometrical complexity. Also in the mid 1800's, J. C. B. St. Venant put forward the question of finding the cross-section of a uniform beam or column that would be most resistant to twisting (the so-called "problem of torsional rigidity"). He conjectured that for a given cross-sectional area, assumed to be a simply-connected region (and with all other physical parameters held fixed), the shape giving the greatest torsional rigidity was the circular one. This problem was finally resolved by George Pólya in 1948 (in the sense that St. Venant had conjectured). Much work has been done on torsion problems since that time, since it is also of interest to consider non-simply connected regions and other variations of the basic problem.

A few years after St. Venant considered the torsion problem, Lord Rayleigh set forth (and formulated conjectures for) (1) the shape of drum that would minimize its fundamental (or "base") tone for fixed area (with other physical parameters held fixed), (2) in static electricity, the shape of capacitor among simply-connected bodies of finite extent that would minimize capacity for given volume, and (3) the shape of clamped plate that would minimize its fundamental frequency for given area. In each case Rayleigh conjectured that the minimizing shape was circular (or spherical, in the case of the 3-dimensional capacitor problem).

Other related problems include the question of what shape minimizes heat loss (described colorfully by Pólya as the explanation for why a cat curls itself into a ball on a cold winter's night) and the shape of a body that minimizes its (gravitational) potential energy.

All of the aforementioned physical problems can be formulated as variational problems, with many leading directly to eigenvalue problems. In the early part of the 20th century there was interesting progress on several of these problems, the most spectacular being the solution of the problem of minimizing the fundamental tone of a drum by Faber and Krahn in independent papers in the early to mid 1920's (the answer is that one should take a circular drum of the given area). Somewhat before Faber and Krahn, Courant had obtained a weaker version of the result, that for fixed perimeter the way to minimize the fundamental tone was to take a circular drum. Earlier Poincaré had made progress on the capacity problem, with the full solution due to Gabor Szegő coming in 1930.

Around 1950, Pólya and Szegő took on the job of studying and systematizing prior works on physical isoperimetric problems, and of advancing the field on a wide front. Their book *Isoperimetric Inequalities in Mathematical Physics*, published at that time, is a classic of the field. The techniques that they put at the forefront included Steiner symmetrization, and, generally, rearrangement inequalities. It could quite justifiably be said that all modern work on isoperimetric inequalities for physical quantities builds on the work of Pólya and Szegő and their collaborators. Pólya and Szegő's book contains, for example, the solutions to the St. Venant and capacity problems mentioned above.

Pólya and Szegő's interest in the subject stimulated interest by others and led to many important and interesting developments in the field. Perhaps foremost among the early contributors to these developments are Payne, Hersch, and Weinberger, who participated in many of the advances and inspired their students and others to enter the field. Thus we find Payne, Pólya, and Weinberger obtaining very simple and nice universal inequalities for combinations of eigenvalues in the mid-50's, and conjecturing what the sharp forms of certain of these inequalities might be. This leads one into the subject of isoperimetric inequalities for eigenvalue ratios, which attracted considerable interest (particularly the ratio  $\lambda_2/\lambda_1$ ) and was finally solved by Ashbaugh and Benguria in 1990. Following a significant advance in work of H. C. Yang in the early 90's, the subject of universal eigenvalue inequalities has taken off, with many papers contributing to and advancing the subject, and with much work continuing to the present day. The work of Yang has allowed researchers to make fundamental connections between the field of universal eigenvalue inequalities and the subject of eigenvalue asymptotics, as begun by Hermann Weyl around 1910. This, too, is a burgeoning field, with key recent contributors including Q. M. Cheng and H. C. Yang, E. M. Harrell and L. Hermi, E. M. Harrell and J. Stubbe, and several others.

Conformal methods play an important role in the study of two-dimensional problems. Szegő used them to prove that the disk minimizes  $\mu_1(\Omega)^{-1} + \mu_2(\Omega)^{-1}$  in the class of simply connected planar domains of given area (here the  $\mu_j$ 's are the positive eigenvalues of the Neumann Laplacian). Hersch proved that the smallest positive eigenvalue of the Laplace–Beltrami operator on a two-sphere cannot exceed the one of the operator for the round metric of the same area. The crucial observation is that the numerator in the Rayleigh quotient,  $\int |\nabla u|^2 dx$ , is conformally invariant when the dimension equals 2. P. C. Yang and S.-T. Yau proved that the first positive eigenvalue on a surface of genus g of given area has an upper bound; moreover, they gave a precise bound. In the case g = 2, Jacobson, Levitin, Nadirashvili, Nigam, and Polterovich proved that Yang and Yau's bound is sharp, and it is saturated on a singular metric on a surface of conformal type of  $y^2 = x^5 - x$ . Their proof relied on some numerics. It would therefore be interesting to have a numerics-free proof. In the case when the dimension of a manifold is higher than 2, Urakawa proved that, in the class of metrics of fixed volume, the first positive eigenvalue of the Laplacian can be arbitrarily large. However, within a given conformal class, it is bounded, and these upper bounds are bounded from below when one varies conformal classes (Friedlander, Nadirashvili). Recently, Colbois, Dryden, and El Soufi studied bounds for eigenvalues of the Laplacian for *G*-invariant metrics in a certain conformal class. Here *G* is a Lie group acting on a manifold.

Obviously there are many other topics that figure in the history of isoperimetric problems and related areas and the most we could do here was point out some of the highlights. To help make up for the deficiencies of such coverage, we conclude with a brief summary of some of the relevant literature, which it is hoped can be used to widen the coverage and give hints of other worthy topics in the general area. For historical orientation, we recommend the article by Lord Kelvin and the expository book by Hildebrandt and Tromba (both mentioned earlier).

For further background on the classical isoperimetric problem one cannot do better than to consult the book of Burago and Zalgaller, *Geometric Inequalities*, and the 1978 review article in the Bulletin of the American Mathematical Society by Robert Osserman, "The isoperimetric inequality." Other books and articles of interest include Tikhomirov's *Stories of Maxima and Minima*, Pólya's *Mathematics and Plausible Reasoning* (in 2 vols.; the most relevant sections of this can be found at the conference website, http://math.arizona.edu/~dido/polya1954.html), and Pólya's article "Circle, sphere, symmetrization, and some classical physical problems", or D. Pedoe's *Circles: A Mathematical View* and N. Kazarinoff's *Geometric Inequalities*.

For aspects of the isoperimetric problem occurring in the setting of Riemannian geometry one can consult the books of Chavel (*Eigenvalues in Riemannian Geometry, Riemannian Geometry: A Modern Introduction*, and *Isoperimetric Inequalities: Differential Geometric and Analytic Perspectives*) and of Marcel Berger (*A Panoramic View of Riemannian Geometry*; or see his books *Geometry I and II* for much useful related information, mostly in the classical setting). Chavel's book *Eigenvalues in Riemannian Geometry* includes topics that extend well into the domain of isoperimetric inequalities for physical quantities.

For modern developments in minimal surface theory and much more, one cannot do better than to consult Almgren's *Plateau's Problem: An Invitation to Varifold Geometry*, and Morgan's *Geometric Measure Theory: A Beginner's Guide* (fourth edition, 2009). Beyond that one has Federer's classic *Geometric Measure Theory*. For the more classical background in minimal surface theory, there are a number of books and articles, among which we mention Osserman's *Survey of Minimal Surfaces* (updated edition, Dover, 1986).

On the side of isoperimetric inequalities for physical quantities one can find much of interest in the works of Pólya and Chavel already mentioned. In the 1960's and beyond, a key role was filled by Payne's SIAM Review paper, "Isoperimetric inequalities and their applications." This paper provides the background and setting for many physical isoperimetric problems (including their mathematical formulation), and also states a variety of open problems and conjectures. In 1991, Payne updated his discussion of many of these problems in his contribution "Some comments on the past fifty years of isoperimetric inequalities" to the book *Inequalities: Fifty Years on from Hardy, Littlewood and Pólya*, edited by W. N. Everitt. Beyond that, one has the books of C. Bandle (*Isoperimetric Inequalities and Applications*), R. Sperb (*Maximum Principles and Their Applications*), and B. Kawohl (*Rearrangements and Convexity of Level Sets in PDE*), dating to the early to mid 80's, and the more recent books of D. Bucur and G. Buttazzo (*Variational Methods in Shape Optimization Problems*), A. Henrot (*Extremum Problems for Eigenvalues of Elliptic Operators*), and S. Kesavan (*Symmetrization and Applications*).

Finally, we mention the excellent book by Lieb and Loss, *Analysis*, second edition, which covers much of interest in the field of symmetrization and rearrangements in the context of the classical inequalities of analysis and mathematical physics, as well as much else besides. In particular, the book covers the problems of minimizing capacity and gravitational potential energy, and has a full discussion of Lieb–Thirring inequalities and their relation to the question of the stability of matter.

### SCHEDULE

#### Intensive Courses at Tunis Science City

#### Friday, May 21, 2010

08:30-09:40 Registration and Breakfast at Tunis Science City 09:40-10:00 Welcome by the Director of the Tunis Science City 10:00-10:50 Rafael Benguria 11:00-11:50 Bruno Colbois 12:00-12:50 Olivier Druet 13:00-14:30 Lunch at Tunis Science City 14:30-15:20 Rafael Benguria 15:30-16:20 Bruno Colbois 16:20-16:45 Coffee Break 16:45-17:35 Olivier Druet 20:00 Dinner

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Saturday, May 22, 2010 09:30-10:20 Bruno Colbois 10:30-10:50 Coffee Break 10:50-11:40 Olivier Druet 11:50-12:40 Rafael Benguria 12:50-14:30 Lunch at Tunis Science City 14:30-15:20 Bruno Colbois 15:30-16:20 Olivier Druet 16:20-16:45 Coffee Break 16:45-17:35 Rafael Benguria 20:00 Dinner

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Intermission Sunday, May 23, 2010: Visit to Bardo Museum (optional)

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### International Conference

Monday, May 24, 2010 Morning Session Presided by: Lotfi Hermi and Najoua Gamara Afternoon Session I Presided by: Mark Ashbaugh Afternoon Session II Presided by: Ahmad El Soufi 08:30-09:15 Registration 09:15-10:30 Opening Ceremony Speech by the Honorable Béchir Tekkari, Minister of Higher Education and Scientific Research 10:30-10:45 Coffee Break 10:45-11:25 M. Hassine Fantar 11:25-11:35 Questions 11:35-12:15 Frank Morgan 12:20-12:40 Tom Carroll 12:45-13:05 Fethi Mahmoudi 13:15-14:30 Lunch Break 14:30-15:10 Evans Harrell 15:20-16:00 Catherine Bandle 16:10-16:30 Coffee Break 16:30-16:45 Inès Ben Omrane 16:50-17:05 Semra Demirel 17:10-17:25 Hakima Bouhadjera 17:30-17:45 Laura Chasman 17:50-18:05 Alina Tukhvatullina 18:10-18:25 Anisa Chorwadwala 18:30-18:45 Tatiana Odzijewicz 20:00-21:30 Dinner

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#### Tuesday, May 25, 2010

Morning Session Presided by: Rafael Benguria Afternoon Session I Presided by: Mohamed Sifi Afternoon Session II Presided by: Hamdi Zorgati 09:00-09:40 Andrea Cianchi 09:50-10:30 Moez Khenissi 10:40-10:55 Coffee Break 10:55-11:35 Jacqueline Fleckinger 11:45-12:05 Bodo Dittmar 12:10-12:30 Antonio Cañete 12:35-12:55 Mourad Zerai 13:00-14:30 Lunch Break 14:30-15:10 Ahmad El Soufi 15:20-16:00 Mohammad Ghomi 16:10-16:30 Coffee Break 16:30-16:45 Rustem Salakhudinov 16:50-17:05 Hassan Ait Haddou 17:10-17:25 Asma Jbilou 17:30-17:45 Khaled Zennir 17:50-18:05 Denis Abramov 18:10-18:25 Lamia Bel Kenani Toukabri 18:30-18:45 Mihai Pascu 20:00-21:30 Dinner

#### 

#### Wednesday, May 26, 2010

Morning Session Presided by: M. Nejib Lazhari and Najoua Gamara 09:00-09:40 Nabila Torki-Hamza 09:50-10:30 Rodrigo Bañuelos 10:40-10:55 Coffee Break 10:55-11:35 Thomas Hoffmann-Ostenhof 11:45-12:25 Vincenzo Ferone 12:35-12:55 Jesse Ratzkin 13:00-13:15 Pedro Solorzano 13:35-15:00 Lunch Break 15:00-18:00 Excursion Carthage Peninsula 20:00-22:30 Culture Ministry Dinner + Concert

#### 

**Thursday, May 27, 2010** Morning Session Presided by: Khalifa Trimèche Afternoon Session I Presided by: Lennie Friedlander Afternoon Session II Presided by: Bruno Colbois

09:00-09:40 Ari Laptev 09:50-10:30 Mohamed Majdoub 10:40-10:55 Coffee Break 10:55-11:35 Richard Laugesen 11:45-12:05 Bartlomiej Siudeja 12:10-12:30 Farit Avkhadiev 12:35-12:55 Alexandre Girouard 13:00-14:30 Lunch Break 14:30-15:10 Gary Lawlor 15:20-16:00 Carlo Morpurgo 16:10-16:30 Coffee Break 16:30-16:45 Luigi Fontana 16:50-17:05 Habiba Guemri 17:10-17:25 Donald Sampson 17:30-17:45 Rebecca Dorff 17:50-18:05 Salem Eljazi 18:10-18:25 Na Wei 18:30-18:45 Samah Horrigue 20:00-21:30 Dinner

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#### Friday, May 28, 2010

Morning Session Presided by: Jacqueline Fleckinger Afternoon Session I Presided by: Habib Bouchriha Afternoon Session II Presided by: Houcine Chebli

09:00-09:40 Hichem Chtioui 09:50-10:30 Frank Pacard 10:40-10:55 Coffee Break 10:55-11:35 Antoine Henrot 11:45-12:05 Ali Saada 12:10-12:30 Leander Geisinger 12:35-12:55 Ryan Hynd 13:00-14:30 Lunch Break 14:30-15:10 Mark Ashbaugh 15:20-16:00 Gerard Philippin 16:10-16:30 Coffee Break 16:30-16:45 Ali Ben Amor 16:50-17:05 Chiara Bianchini 17:10-17:25 Brahim Dridi 17:30-17:45 Amine El Amri 17:50-18:05 Ilnar Shafigullin 18:10-18:25 Dinara Giniyatova 18:30-18:45 Sidi Mohammed Bahri 20:00-21:30 Dinner

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#### Saturday, May 29, 2010

Session Presided by: Evans Harrell 09:00-09:40 Nabil Gmati 09:50-10:30 Rupert Frank 10:40-10:55 Coffee Break 10:55-11:35 Almut Burchard 11:45-12:05 Mohamed Ben Rhouma 12:10-12:30 Andrea Mondino 12:35-12:50 Salah Boulaaras 12:55-13:10 Mette Iversen 13:15-13:30 End of Conference

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Sunday, May 30, 2010: Excursion Triangle Sousse, El Jem and Kairouan

#### 



## THE HISTORY DESCRIPTION OF AFRICA

AL-HASSAN IBN-MOHAMMED AL-WEZAZ AL-FASI

A MOOR, BAPTISED AS GIOVANNI LEONE, BUT BETTER KNOWN AS LEO AFRICANUS 1494-1554

DONE INTO ENGLISH IN THE YEAR 1600,

### JOHN PORY

#### A description of the mightie citie of Tunis.



His citie is called by the Latines Tunetum, and by the Arabians Tunus, which name they thinke to be corrupt, because it signifieth nought in their language: but in olde time it was called Tarsis, after the name of a citie in Asia. At the first it was a small

towne built by the Africans vpon a certaine lake, about twelue miles from the Mediterran sea. And vpon the decay of Carthage Tunis began to increase both in buildings and inhabitants. For the inhabitants of Carthage were loth to remaine any longer in their owne towne, fearing least some armie would have beene sent out of Europe : wherefore they repaired vnto Tunis, and greatly enlarged the buildings thereof.





## **{THAGE** ERNEST BABELON PARIS 1896

Plusieurs milliers de citoyens s'y embarquèrent en toute hâte et prirent la mer pour aller fonder sur une terre plus hospitalière une nouvelle Tyr, sous la conduite d'Élissar, à qui cette migration forisée valut le surnom de Didon « la fugitive ». Les émigrants se dirigèrent vers la côte d'Afrique où les établissements tyriens étaient nombreux et où ils étaient assurés de trouver des compatriotes, des parents mêmes, disposés à les bien accueillir. Élissar ... acheta, moyennant une redevance annuelle, un vaste terrain où fut bâtie « la ville neuve », Kart-hadschat : c'est ce nom dont les Grecs, par corruption, ont fait Carchedon, et les Romains, Carthage. Didon, poursuit la légende, ayant obtenu des Liby-Phéniciens, pour elle et ses compagnons, autant de terre qu'il en peut tenir dans la peau d'un bœuf, immola un de ces animaux, puis elle en découpa la peau en lanières assez étroites pour en envelopper toute la colline qui, de là, prit le nom de Byrsa. Il est aisé de deviner que la dernière partie de ce récit ne repose que sur un jeu de mots : le rapprochement du mot grec βύρσα, qui signifie cuir, peau, et du mot phénicien bosra ( / ງາς ) dont le sens est citadelle, forteresse. Tout n'est pourtant pas à dédaigner dans le récit de l'hégire d'Élissar, dont le vieux Caton, Trogue-Pompée, saint Augustin se sont fait l'écho, tandis que Virgile donnait à Didon l'immortelle consécration de sa poésie.

## KEYNOTE SPEAKERS



Catherine Bandle University of Basel Switzerland c.bandle@gmx.ch www/math.unibas.ch/~bandle

Catherine Bandle studied mathematics at the ETH in Zürich where she obtained all her degrees. After several years as a Postdoctoral Fellow at the Carnegie-Mellon and the Stanford Universities, and at Research Institutes in Geneva and Zürich, she became Professor at the University of Basel. Since 2003 she retired. Her interests are nonlinear elliptic and parabolic equations and calculus of variations. She has been visiting professor and scientist in many European countries, in the US, in Israel, Chile and Hong Kong.

#### Title: Best Sobolev constants with critical growth on spheres

Abstract: Upper and lower bounds for the best Sobolev constant of an arbitrary domain on a *N*-dimensional sphere are derived. In the case of a geodesic ball its value will be determined explicitly. For small balls the Sobolev constant is not attained and does not depend on the radius. The proof uses Pohozaev type identities and the concentration-compactness principle. The talk is based on joint work with L. A. Peletier and S. Stingelin.





Rodrigo Bañuelos Purdue University USA banuelos@math.purdue.edu www.math.purdue.edu/~banuelos

Rodrigo Bañuelos earned his Ph.D. in Mathematics from the University of California, Los Angeles in 1984. He spent two years as a Bantrell Research Postdoctoral Fellow at the California Institute of Technology and one year as a National Science Foundation Postdoctoral Fellow at the University of Illinois at Urbana before moving to Purdue University in 1987. He has served as Head of the Department of Mathematics since 2007.

Bañuelos' research interests lie at the interface of probability, harmonic analysis and spectral theory. He was a National Science Foundation Presidential Young Investigator from 1989 to 1994. He was elected Fellow of the Institute of Mathematical Statistics in 2003 and received the Blackwell-Tapia National Prize in Mathematics in 2004. From 1998 to 2002, he served on the Scientific Advisory Council of the Mathematical Science Research Institute at Berkeley and was a member of the United States National Committee on Mathematics from 1998 to 2001. He served on the Board of Trustees of the Institute for Pure and Applied Mathematics at UCLA 2005-2009. He has served on several editorial boards of mathematical journals, including the Annals of Probability, the Transactions and Memoirs of the American Mathematical Society and Revista Matemática Iberoamericana.

#### Title: Isoperimetric inequalities for Lévy processes

Abstract: We shall discuss how several isoperimetric inequalities for the Laplacian, when viewed in terms of Brownian motion, extend to a wide class of Lévy processes. The results are derived from the Brascamp-Lieb-Luttinger inequalities but the probabilistic structure of the processes coming from the Lévy-Khintchine formula plays an important role in the proofs.





Rafael Benguria P. Universidad Católica de Chile Chile rafael.benguria@googlemail.com www.fis.puc.cl/~rbenguri

Rafael Benguria was born in Santiago de Chile. He did his undergraduate at the Universidad de Chile where he graduated as an Electrical Engineer in 1974. In 1975 he got his Master in Physics from the Universidad de Chile. He got his Ph.D. in Physics from Princeton University in 1979. His advisor was Professor Elliott Lieb. After spending two years as a postdoctoral research associate at Rockefeller University in NY, he went back to Chile in 1981. From 1981 to 1990 he was at the Physics Department of the U. de Chile, where he became a full professor in 1987. In 1990 he joined the faculty of Physics at the P. Universidad Catolica de Chile. He works in Mathematical Physics. His areas of research are Spectral Geometry, Stability of Matter related problems, Nonlinear Elliptic Equations, and Reaction-Diffusion Equations. He is a member of the Academia Chilena de Ciencias, and he got the Premio Nacional de Ciencias (Chile) in 2005. He is a member of the IAMP (International Association of Mathematical Physics). He served as Secretary (1987-1989) and later as President (1989-1991) of the Chilean Physical Society.

Intensive Course: Isoperimetric Inequalities for Eigenvalues of the Laplacian

Abstract: Please refer to the "Intensive Courses" section of this booklet.



Almut Burchard University of Toronto Canada almut@math.toronto.edu www.math.toronto.edu/almut

Almut Burchard completed her basic studies in Heidelberg, Germany, and earned Ph.D. degree from the Georgia Institute of Technology in 1994. Her doctoral thesis, on the equality cases in Riesz' rearrangement inequality, was supervised by Michael Loss. After several years in Princeton, she joined the University of Virginia in 1998, and moved to the University of Toronto in 2005.

Almut has worked on a variety of problems in Analysis, including sharp geometric inequalities, evolution equations, and critical percolation, as well as problems arising from computer networks, and mathematical biology. Her current interests include thin liquid films, random symmetrizations, and rearrangement inequalities for convolution functionals. She received a Sigma Xi Ph.D. thesis award in 1995, and a Sloan Research Fellowship in 1999.

Dr. Burchard received the Sigma Xi Best Ph.D. thesis award at Georgia Tech in 1994, and was awarded a Sloan Research Fellowship in 1999. She is a member of the American Mathematical Society and the International Association of Mathematical Physics.

Title: On the energy-minimizing steady states of a thin film equation

Abstract: In this talk, I will describe recent joint work with Marina Chugunova and Ben Stephens on a degenerate fourth order equation in one spatial variable with periodic boundary conditions. This is a model for the evolution of a thin liquid film on the surface of a horizontal cylinder. The equation formally defines a gradient flow on the space of nonnegative functions of a given mass, and one may suspect that minimizers of the corresponding energy should attract most solutions.

We show that for every given mass, the energy has a unique minimizer, which (depending on the mass) may be strictly positive, or else meets the dry region at zero contact angle. While positive minimizers are exponentially attractive, the distance to a steady state with a dry region cannot decay faster than a power law.





Bruno Colbois Université de Neuchâtel Suisse Bruno.Colbois@unine.ch www3.unine.ch/bruno.colbois

Bruno Colbois obtained his PhD in mathematics at the University of Lausanne in 1987 working with Peter Buser. Assistant professor at the Polytechnic School of Zürich (ETHZ) (1992-94), Professor at the Université de Savoie (1994-2000) and since 2000, Professor at Université de Neuchâtel, where he has served as Department Head, 2004-2008. For the years 2010 and 2011, Bruno Colbois is President of the Swiss Mathematical Society (www.math.ch).

His research interest is in the geometric approach of spectral theory, and he has around 40 publications on the subject. He has supervised or co-supervised 10 Ph.D theses.

Intensive Course: Laplacian on Riemannian Manifolds

Abstract: Please refer to the "Intensive Courses" section of this booklet.



Andrea Cianchi Dipartimento di Matematica "U. Dini", Università di Firenze Italy cianchi@unifi.it www.math.unifi.it

Born in Florence (Italy) in 1963, received his Ph.D. in Mathematics from the University of Florence. Professor of Mathematical Analysis at the University of Florence. Head of the Department of Mathematics and Applications to Architecture 2006–2008. Principal Investigator of National Research Projects of the Italian Ministry of University and Research 2006–2008 and 2008–2011. Author of about seventy-five research papers on quantitative and qualitative properties of solutions to elliptic PDEs, inequalities of Sobolev type, isoperimetric and isocapacitary inequalities, fine properties of symmetrizations and rearrangements.

Title: Isocapacitary vs. isoperimetric methods in elliptic PDEs

Abstract: We shall discuss several questions, such as existence, uniqueness and regularity of solutions to elliptic boundary value problems affected by irregularities in the domain and/or in the data, via methods relying upon either isoperimetric or isocapacitary inequalities. Eigenvalue problems will also be focused.



Olivier Druet École Normale Supérieure de Lyon, France Olivier.Druet@umpa.ens-lyon.fr

Olivier Druet obtained his PhD in 2001 at the University of Cergy-Pontoise. His thesis Asymptotiques géometriques, phénomènes de concentration et inégalités optimales was supervised by Professor Emmanuel Hebey. Since 2002, he has been "Chargé de Recherches" at C.N.R.S. in a position at the Ecole Normale Supérieure de Lyon. He also spent one year at PIMS in Vancouver in 2007-2008. Intensive Course: Isoperimetric inequalities in negatively curved spaces

Abstract: Please refer to the "Intensive Courses" section of this booklet.

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Ahmad El Soufi Université François Rabelais de Tours, France elsoufi@univ-tours.fr www.lmpt.univ-tours.fr/~elsoufi

Ahmad El Soufi was born in Lebanon in 1960. He arrived in France in 1977 to pursue university studies. El Soufi received his doctorat de 3ième cycle (1983) and doctorat d'état (1992) from the Fourier Institute, Grenoble University, working under the supervision of Prof. Sylvestre Gallot. He is a full professor at François Rabelais University of Tours since 1993. From 1998 to 2002 he was Director of the GDR (Groupement De Recherche) of CNRS called "Minimal Surfaces and Harmonic Maps". Since 2007, he has been Regional Scientific Officer at the International Center for Pure and Applied Mathematics ICPAM-CIMPA, in charge of Mediterranean rim countries. He supervised 8 Ph.D. theses and authored about 45 scientific publications.

## $\label{eq:title: Isoperimetric control of the eigenvalues of the Laplace-Beltrami operator, and of the Steklov problem on a compact hypersurface$

Abstract: Reilly (1977) and Chavel (1978) have proved independently that for a compact orientable hypersurface  $\Sigma$  of the Euclidean space, the first positive eigenvalue, normalized by the volume, is bounded above in terms of the isoperimetric ratio of the domain bounded by  $\Sigma$ .

In a recent work in collaboration with Bruno Colbois and Alexandre Girouard, we establish inequalities of Reilly-Chavel type for all the higher order eigenvalues, i.e., the isoperimetric ratio allows a control of the entire spectrum of a hypersurface. Our results extend to the case where the ambient space is not Euclidean but a complete Riemannian manifold satisfying certain hypotheses.

In a second part of the talk, we will discuss the eigenvalues of the Steklov problem and show how these eigenvalues can be controlled, in dimension 3 and larger, in terms of the inverse of the isoperimetric ratio. The 2-dimensional case is of particular interest and will be discussed separately.





M'hammed Hassine Fantar Titulaire de La Chaire du Président Ben Ali Tunisia mh.fantar@utm.rnu.tn www.chairebenali.tn

Alumnus of the Collège Sadiki and the École Normale Supérieure de Tunis, Professor Fantar completed his Etudes de 3ième Cycle at the Université de Strasbourg and Université de la Sorbonne in Paris, specialized in West-Semitic languages and the Civilizations of the Middle East, with emphasis on Phoenicia and Carthage. Former General Director of the National Institute of Archaeology and Art of Tunis (1982-1987), Director of Research at the National Institute of Patrimony of Tunis, director of the Center for the Study of Phoenician and Punic Civilization and Libyc Antiquities and founder of the Reppal review, director of the Division of Museums at the National Institute of Patrimony, and Professor of Ancient History, Archaeology, and History of Religions at the University of Tunis. Visiting Professor at the universities of Rome, Bologna, Cagliari, Tripoli, Benghazi, Collège de France and Leuven. Professor Fantar holds a Doctor honoris causa from the Università di Bologna, officier de l'ordre du mérite culturel de France, officier de l'ordre de la République Tunisienne, and officier de l'ordre du mérite de la Republique Italienne, and Commandeur de l'ordre du mérite pour l'Enseignement Supérieur et la Recherche Scientifique. Founding member of the National Foundation for the Establishment of Texts and Translations, member of the National Cultural Committee, corresponding member of the Archaeological Institute of Berlin, council member of the Arab House of the Book, member of the International Association for the Study of Mediterranean Cultures, member of the Accademia Nazionale dei Lincei in Rome, corresponding member of Instituto Italiano per l'Africa e l'Oriente, member of the Arab Union of Historians and its representative in Tunisia, member of the National Union of Tunisian Writers, member of the Higher Committee for Human Rights and Fundamental Liberties, and Vice President of the Hannibal Club. Professor Fantar is Lauréat de l'Academie des Inscriptions et Belle Lettres, and has received

the Prix Toutain Blanchet for his book, *Kerkouane, une cité punique du Cap Bon*. He also received the Prix du Comité Culturel National for his book *Jugurtha* (in Arabic), and the Prix Sabatino Moscati for the Mediterranean in 2002.

Since 2002, Professor Fantar held the "Chair Ben Ali for Dialogue of Civilizations and Religions", an academic outfit which promotes tolerance, solidarity and dialogue through various intellectual and scientific activities.

Title: Of the Myth of Elyssa-Dido and her historical and cultural significance for the Maghreb and the whole of the Mediterranean

Abstract: Mon intervention traiterait du mythe d'Elyssa-Didon de sa portée historique et culturelle tant pour le Maghreb que pour l'ensemble de la Méditerranée.



Vincenzo Ferone Università degli Studi di Napoli "Federico II", Italy ferone@unina.it www.dma.unina.it/ferone

Graduated in Physics at the Università di Napoli (1984). Ph.D. in Applied Mathematics and Informatics at the Università di Napoli (1990). "Professore ordinario" of Mathematical Analysis at the Faculty of Engineering of the Università di Napoli "Federico II" (from November 2000). Visiting professor at various universities: Université de Paris IX-Dauphine (1995), Université de Poitiers (1998), Université de Paris VI (2004), Australian National University in Canberra (2005). Several invitations to give lectures in meetings (Oberwolfach, Nagano, Berlin, Roma, Bourges, etc.) and at universities and institutions (College de France, Versailles, Heidelberg, Colonia, Kyoto, Bologna, Firenze, Roma, etc.); main speaker at XVIII Congresso dell'Unione Matematica Italiana, Bari (2008). Awards: "Premio per le Scienze Matematiche" of Accademia di Scienze Fisiche e Matematiche della Società Nazionale di Scienze, Lettere e Arti in Napoli (1992); Premio "Tenore" per l'Analisi Matematica of Accademia Pontaniana (2006). Member of the "Consiglio di Redazione" of the journal "Ricerche di Matematica". Congress organization: "Elliptic and Parabolic P.D.E.'s and Applications", Capri (1994); "Equazioni differenziali: metodi analitici, geometrici e funzionali, e applicazioni", Napoli (2000); "Aspetti teorici e applicativi di equazioni alle derivate parziali", Maiori (2004); "Mathematical Aspects of Mechanics - An international meeting dedicated to the eightieth birthday of Grigory Isaakovich Barenblatt", Napoli (2007); Summer School "Topics in Partial Differential Equations", Napoli (2008). Main research interests: optimization problems on classes of functions with prescribed rearrangement, comparison results for solutions of partial differential equations, minimization of functionals, existence and regularity for solutions of non linear elliptic and parabolic equations.

#### Title: Sharp isoperimetric inequalities in the plane

Abstract: The classical isoperimetric inequality in the plane states that, among all the subsets of  $\mathbb{R}^2$  of prescribed finite measure, the disk has the smallest perimeter.

Bonnesen (1921, 1924) introduced some remarkable inequalities which imply the isoperimetric one. If the isoperimetric deficit  $\Delta P(E)$  of a set E is defined as the difference between the perimeter of E and the perimeter of the disk with the same measure, Bonnesen's inequalities state that  $\Delta P(E)$  is bounded from below by some quantity which vanishes only when E is a disk. Later Bonnesen's work led to the study of a wider class of inequalities nowadays known as Bonnesen–style isoperimetric inequalities.

We discuss the situations where the bound from below of  $\Delta P(E)$  is given in terms of the so-called *Fraenkel* asymmetry index or through the Hausdorff distance from a ball. For convex sets of the plane it is possible to prove a sharp form of such inequalities and the optimal sets can be given.

The results described above can be found in two joint papers with Angelo Alvino and Carlo Nitsch.





Jacqueline Fleckinger Université Toulouse 1 France jfleckinger@gmail.com math2.univ-tlse1.fr/~jfleck

- 1968-1970: First steps in Tunisia. First position as Assistante Agrégée in Tunis. She attended the PDE seminar led by MS Baouendi and studied the famous Mark Kac paper.
- 1970-1986: Maître de Conférence at Institut Polytechnique in Toulouse; Several results on the counting function.
- 1986-2008: Professor at University Toulouse 1; Studies on the role of eigenvalues on the solutions to some non linear PDE.
- Officier Palmes Académiques
- Medaille de la Société Mathématique de Tunisie.
- Coeditor Journal Egypt. Math. Soc. (JEMS) and Adv. Pure and Applied Math (APAM)

#### Title: Around eigenvalues and eigenfunctions

Abstract: "Can one hear the shape of a drum?" was the first paper that I studied (it was shown to me by Jacques Faraut in Tunis).

In this talk we recall some results on the "counting function" (estimates of large eigenvalues) for classical problems, problems defined on unbounded domains and weighted elliptic operators. We also recall results for problems defined on irregular domains (domains with fractal boundaries).

Then we study the role of eigenvalues for some elliptic problems and their influence on the nodal domains of the solutions.



Rupert Frank Princeton University USA rlfrank@math.princeton.edu www.math.princeton.edu/~rlfrank

Rupert Frank received his PhD in 2007 from the Royal Institute of Technology in Stockholm, Sweden. His thesis 'Hardy-Lieb-Thirring inequalities for eigenvalues of Schrödinger operators' was directed by Ari Laptev. Since 2007 he is a member of the mathematical physics group around Elliott Lieb at Princeton University, where he is appointed as post-doc, instructor and assistant professor.

Rupert Frank's research is centered on questions from analysis, spectral theory and mathematical physics, and has been recognized by the Young Scientist Prize in Mathematical Physics of the International Union of Pure and Applied Physics.

#### Title: Stability and absence of binding for multi-polaron systems

Abstract: We prove two facts about the binding of polarons, or its absence, which have been longstanding problems. First, the transition from many-body collapse to the existence of a thermodynamic limit for N polarons occurs precisely at  $U = 2\alpha$ , where U is the electronic Coulomb repulsion and  $\alpha$  is the polaron coupling constant. Second, if U is large enough, there is no multi-polaron binding of any kind. While these results are valid for quantized fields (the Froehlich model), in the talk we will focus on the technically easier case of classical fields (the Pekar-Tomasevich model).

(The talk is based on joint work with E. H. Lieb, R. Seiringer and L. E. Thomas.)

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Mohammad Ghomi Georgia Institute of Technology USA ghomi@math.arizona.edu people.math.gatech.edu/~ghomi

Mohammad Ghomi is a geometer and professor of Mathematics at Georgia Institute of Technology. He completed his PhD thesis in 1998 under the direction of Joel Spruck at Johns Hopkins University, and taught at University of California at Santa Cruz, University of South Carolina, and Penn State University, before moving to Atlanta in 2005. Mohammad Ghomi's research centers around differential geometry and topology of Riemannian submanifolds, specially curves and surfaces in Euclidean space and convexity problems. He has a wide range of interests in this area including analytic, algebraic, and discrete aspects of curve and surface theory to which he has contributed in over two dozen publications to date. He was a recipient of an NSF CAREER award in 2003

#### Title: Relative isoperimetric inequality outside convex bodies

Abstract: We prove that the area of a hypersurface which traps a given volume outside of a convex body in Euclidean *n*-space must be greater than or equal to the area of a hemisphere trapping the given volume on one side of a hyperplane. This result generalizes the classical isoperimteric inequality. The proof is based on a sharp estimate for total positive curvature of hyperurfaces whose boundary lies on a convex body and meets that body orthogonally from the outside. We will also discuss some possible generalizations to capillary surfaces. This is joint work with Jaigyoung Choe and Manuel Ritoré.

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Antoine Henrot Federation Charles Hermite, Nancy, France antoine.henrot@mines.inpl-nancy.fr www.iecn.u-nancy.fr/~henrot

Antoine Henrot received a PhD in Algebraic Topology and a HDR in Partial Differential Equations from Nancy, France. He had been Assistant Professor, then Maître de Conférences (associate professor) at the University of Nancy I from 1984 to 1992. He was promoted to Full Professor at the University of Besançon, France where he spent the years 1992-1998. He moved back to Nancy in 1998 to the Ecole des Mines and Institut Elie Cartan, Nancy. There he served as Head of the Institut Elie Cartan (Math Department of Nancy, 120 people) from 2005 to 2008. He is currently Head of the Federation Charles Hermite of Nancy and serves as expert for different institutions in France and Italy.

Henrot wrote two books: "Variation et Optimisation de Formes" (with Michel Pierre, 2005), and "Extremum Problems for Eigenvalues of Elliptic Operators" (2006). He is the author of more than 50 papers in international journals. He supervised the PhD dissertations of 10 students who work now in academic institutions or in the industry, and has organized some important conferences in Luminy (France), Oberwolfach (Germany), Vittel (France).

#### Title: Some shape optimization problems with polygonal solutions

Abstract: In this talk, we will consider four optimization problems,

- the farthest convex set (joint work with Evans Harrell)
- the largest first eigenvalue for the Laplacian with Dirichlet boundary conditions among convex sets with some geometric constraints
- a geometric functional involving area and perimeter
- the Mahler functional. (Joint work with Evans Harrell and Jimmy Lamboley)

All these functionals have optimal solutions which are polygonal. We will prove this fact and give a general method to attack this kind of problem.





Thomas Hoffmann-Ostenhof Erwin Schrödinger Institute and Univ. of Vienna, Austria thoffman@esi.ac.at www.esi.ac.at

Diploma in Chemistry at the ETH Zürich and PhD in Vienna. At the beginning of his scientifc activities he investigated mostly problems stemming from Theoretical Chemistry. Gradually in collaboration with Maria Hoffmann-Ostenhof (Dept. Mathematics, University of Vienna) he shifted towards Mathematical Physics and problems from analysis. Interests include elliptic PDE (regularity of Coulombic wave functions), spectral problems concerning Schrödinger operators and membranes and recently also nonlinear problems related to spectral theory of membranes.

Since 1993 he is secretary of the Erwin Schrödinger International Institute for Mathematical Physics, (ESI). He has co-organized a few scientific activities, mostly at the ESI.

#### Title: Spectral Minimal Partitions

Abstract: We consider Dirichlet Laplacians on bounded domains mostly in  $\mathbb{R}^2$ . Associated to such a domain we define for each positive integer k a spectral minimal partition. Such a partition is a specific, not necessarily unique, partition of the domain into k pairwise disjoint subsets which satisfy some natural spectral conditions.

It turns out that there is an interesting connection between those spectral minimal partitions and nodal domains of eigenfunctions. Furthermore by analyzing spectral minimal partitions we achieve a characterization of the case when there is equality in Courant's nodal theorem. Some of the problems are related to shape optimization and to isoperimetric problems.

This is joint work with Bernard Helffer, Susanna Terracini and partly with Virginie Bonnaillie-Nöel. The basic problem and some of the results can be found in:

Bernard Helffer, Thomas Hoffmann-Ostenhof, Susanna Terracini. Nodal domains and spectral minimal partitions. Ann. I. H. Poincaré-AN 26, 101-138, 2009.

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Ari Laptev Imperial College, London United Kingdom a.laptev@imperial.ac.uk www2.imperial.ac.uk/~alaptev

Ari Laptev received his PhD in Mathematics from Leningrad University (LU) in 1978, under the supervision of Michael Solomyak. His research interest is Spectral Theory of Differential Operators.

Between 1972-77 and 1977-82 he was employed as a junior researcher and as Assistant Professor at the Mathematics & Mechanics Department of LU. In 1981- 82 he held a post-doc position at the University of Stockholm and in 1982 he lost his position at LU due to his marriage to a British subject. Up until his emigration to England in 1987 he was working as a builder, constructing houses in small villages in the Novgorod district of Russia. In 1987 he was employed in Sweden, first as a lecturer at Linköping University and then from 1992 at the Royal Institute of Technology (KTH). In 1999 he became a professor at KTH and also a Vice Chairman of its Mathematics Department. In 1992 he was granted Swedish citizenship.

A. Laptev was the President of the Swedish Mathematical Society from 2001 to 2003 and the President of the Organizing Committee of the Fourth European Congress of Mathematics in Stockholm in 2004. From January 2007 he has been employed as Professor by Imperial College London and from September 2008 he became the Head of Department of Mathematics.

A. Laptev was elected the President of the European Mathematical Society for the period January 2007-December 2010.

#### Title: On some spectral inequalities for non-elliptic partial differential operators

Abstract: I shall give a proof of some spectral inequalities for the moments of the eigenvalues for a class non-elliptic Partial Differential Operators for which phase volume type estimates do not exist.



Richard Laugesen University of Illinois at Urbana-Champaign, USA laugesen@illinois.edu www.math.uiuc.edu/~laugesen

Richard Laugesen studied for his undergraduate degree in New Zealand, and then moved to the United States for graduate work. His Ph.D. on "Extremal problems involving logarithmic and Green capacity" was received in 1993 under the direction of Professor Albert Baernstein II, at Washington University in St. Louis. After postdoctoral appointments at the University of Michigan, Johns Hopkins University and the Institute for Advanced Study, in 1997 Professor Laugesen joined the Mathematics Department at the University of Illinois at Urbana-Champaign. His recent research focuses on isoperimetric problems for eigenvalues of the Laplacian and bi-Laplacian.

#### Title: The method of the unknown trial function: sharp lower bounds on Laplace eigenvalues

Abstract: Sharp lower bounds on eigenvalues of the Laplacian are difficult to obtain. The most widely studied methods involve rearrangement or symmetrization. These methods tend to fail for anything beyond the first Dirichlet eigenvalue. We show how a different, variational approach can yield sharp lower bounds on sums of Laplace eigenvalues.

The method applies to families of domains obtained by linear transformation. In particular it covers all triangles, parallelograms and ellipses. The eigenvalue bound is obtained by "interpolating" between two domains with known eigenvalues. The (unknown) eigenfunctions of our given domain are linearly transplanted into Unknown Trial Functions on the endpoint domains. The desired lower bound then follows from one or the other of these endpoint estimates - though we cannot tell which!

To illustrate the method, we minimize the first nonzero Neumann eigenvalue on triangles of given diameter, obtaining an optimal Poincaré inequality for triangles. The minimal domain is degenerate isosceles. We also minimize sums of arbitrarily many Dirichlet eigenvalues, among triangles of given diameter, with the minimizer being equilateral. This last result lends support to the Spectral Gap Conjecture for triangles.

[Joint work of R. S. Laugesen and B. A. Siudeja.]



Gary Lawlor Brigham Young University USA lawlor@mathed.byu.edu mathed.byu.edu/~lawlor

Gary Lawlor is a mathematician at Brigham Young University, currently residing in the mathematics education department. Lawlor received his Ph.D. in mathematics in 1988 from Stanford University, working under Frank Morgan and Brian White. He did a postdoc at Princeton University under Fred Almgren, where he was promoted to assistant professor in 1990.

Lawlor has devoted his research career to seeking deeper understanding and simpler proofs for well-known results, and to finding a calibration proof for multiple bubbles. The latter pursuit began in 1989 and continued through twenty years of breakthroughs and setbacks until reaching fulfillment in 2010. The new proofs of known results include the tetrahedral soap film, various Steiner trees in the plane, a number of proofs for

the isoperimetric problem in the plane, a very geometric minimization proof for the brachistochrone, cones over products of spheres, and an elementary proof for the isoperimetric problem in *n*-space.

Another interesting development of Lawlor's on a well-known topic is a multi-variable L'Hospital's rule. New results have also come along the way, including area-minimizing pairs of planes, high-

dimensional soap-film-like minimizers, and a nested family of unorientable area-minimizing surfaces, each of which is the singular set of the next.

#### Title: A metacalibration proof that double bubbles in 3-space are area-minimizing

Abstract: Calibration is a method for proving geometric minimization. As a calibrating function or differential form sweeps across a competing curve or surface, it creates a local comparison of accumulating size versus progress toward meeting the constraints required of all competitors. Some form of the fundamental theorem of calculus or Stokes' theorem regulates the totals of these local quantities, and thereby orchestrates the global comparison.

From the variables available for the calibrating function, classical calibration is sensitive to position in space and tilt of tangent line or plane. This works well when constraints are positional, usually meaning that competitors have fixed boundary. When the competition involves fixed arc length or enclosed area or volume, however, a new type of variable is needed. The new idea of metacalibration can be described as classical calibration combined with slicing, making available variables that are themselves results of integration, such as cross-

sectional area of a slice or volume swept out so far. This enhancement is closely analogous to Fubini's theorem, which adds structure to an unorganized multivariable integral; then the inner integral(s) are enabled to provide cumulative-type quantities as integrands for the outer integral(s).

Metacalibration adds beauty, simplicity and unification to the field of geometric optimization proofs. In this talk we discuss a new proof of the double bubble theorem in Euclidean 3-space.



Mohamed Majdoub Faculty of Sciences of Tunis Tunisia mohamed.majdoub@fst.rnu.tn

Mohamed Majdoub received his PhD in 2000 from the Faculty of Sciences of Tunis, Tunisia. His thesis *Equations d'évolution et influence d'une perturbation sous-critique* was directed by Hajer Bahouri. Since 2001 he is a member of the Partial Differential Equations group at the Faculty of Sciences of Tunis, where he is professor.

Mohamed Majdoub's research is centered on questions from Analysis, Fluid Mechanics, mathematical physics and dispersive equations.

#### Title: On the lack of compactness in the 2D critical Sobolev embedding

Abstract: This presentation is devoted to the description of lack of compactness of  $H^1_{rad}(\mathbb{R}^2)$  in Orlicz space. Our result is expressed in terms of the concentration-type examples derived by P. -L. Lions. The approach we adopt to establish this characterization is completely different from the methods used in the study of the lack of compactness of Sobolev embedding in Lebesgue spaces and take into account the variational aspect of Orlicz spaces. We also investigate the feature of the solutions of non-linear wave equation with exponential growth, where the Orlicz norm plays a decisive role.





Frank Morgan Williams College USA Frank.Morgan@williams.edu math.williams.edu/morgan

Frank Morgan studies optimal shapes and minimal surfaces. He has published over 100 articles and six books, including "Geometric Measure Theory: a Beginner's Guide" and "The Math Chat Book," based on his live, call-in TV show and column. Founder of the NSF "SMALL" Undergraduate Research Project, inaugural winner of the Haimo national teaching award of the Mathematical Association of America, he is Atwell Professor of Mathematics at Williams College. As Vice-President of the American Mathematical Society, he has launched their new Graduate Student Blog.

#### Title: Isoperimetric Problems in Spaces with Density

Abstract: Densities on spaces, which weight volume and perimeter by the same factor, are in many ways more natural than metrics, which scale different dimensions differently. Long valued by probabilists, they have enjoyed a surge in interest since their appearance in Perelman's proof of the Poincaré Conjecture. There has been some recent progress by undergraduates. All welcome; no prerequisites.





Carlo Morpurgo University of Missouri USA morpurgoc@missouri.edu

Carlo Morpurgo obtained his Ph.D. degree in 1993 at Washington University in St. Louis and his advisor was Prof. Albert Baernstein II. Morpurgo held postdoctoral positions at the Università di Milano, Italy and the University of Texas at Austin, as an "R.H. Bing Fellow". He held a faculty position at both the Università di Milano-Bicocca, and in 2000 he joined the Department of Mathematics at the University of Missouri, Columbia.

Morpurgo has done work on: heat kernel estimates; extremal problems for eigenvalues, heat kernel traces and zeta functions; conformally invariant operators; rearrangement inequalities in analysis and probability; sharp Sobolev and log-Sobolev type inequalities. His recent work centers around sharp Moser-Trudinger and Beckner-Onofri inequalities in both Euclidean and CR settings.

#### Title: Adams and Moser-Trudinger inequalities

Abstract: In this talk I will present recent sharp exponential integrability results obtained in joint work with Luigi Fontana.



Frank Pacard Université Paris Est-Créteil Val de Marne, France pacard@univ-paris12.fr http://perso-math.univ-mlv.fr/users/pacard.frank

Education

- 1987: École Polytechnique.
- 1990: École Nationale des Ponts et Chaussées.
- 1991: PhD in Mathématiques. Université Paris 11.
- 1993: Habilitation à Diriger des Recherches en Mathématiques. École Normale Supérieure de Cachan.

Appointments

- 1989-1990 Ingénieur Civil.
- 1990-1994 Research position at the CERGRENE.
- 1994- . . . Professor. Université Paris 12.
- 2004- . . . Member of the l'Institut Universitaire de France.
- 2007-... Chargé de Mission at the French Ministry of Higher Education and Research.

Frank Pacard organized several international conference at MSRI Berkeley (October 2005), CIRM (Centre International de Rencontres Mathématiques) Luminy (June 2007), BIRS Banff International Research Station, Canada (August 2007), ICTP Trieste, Italy (June 2008) and Beijing (July 2009). He has supervised the PhD theses of 9 students.

#### Title: Constant mean curvature surfaces in Riemannian manifolds

Abstract: I will give an overview of recent progress in the construction and understanding of constant mean curvature surfaces (or hypersurfaces) in a given Riemannian manifold equipped with a generic metric.



Nabila Torki-Hamza Université 7 Novembre à Carthage, Faculté des Sciences de Bizerte, Tunisia natorki@yahoo.fr www.univ7nc.rnu.tn

Nabila Torki-Hamza earned a degree in Mathematics from Ecole Normale Supérieure de Bizerte (Tunisia 1984). She then received a Ph.D. from Université Joseph Fourier de Grenoble (France 1989) working with Professor Yves Colin de Verdière on "Stabilité des valeurs propres et champs magnétique sur une variété Riemannienne et sur un graphe". She has been teaching since 1989 in Faculté des Sciences de Bizerte (which had been Ecole Normale Supérieure de Bizerte until 1993). She served as Head of the Mathematics Department from 1999 to 2002, and was a member of the Scientific Committee for the periods of 1996-2002 and 2005-2008.

Her research interests lie at the interface of spectral theory, spectral graph theory and Riemannian geometry. Since 2008, she has been working with Yves Colin de Verdière and Françoise Truc on combinatorial Schrödinger operators on infinite graphs looking for the analogs of results known for Riemannian manifolds.

#### Title: Infinite graph Laplacians

Abstract: We would like to extend to infinite graphs some results about essentially self-adjointness for Schrödinger operators on Riemannian manifolds. We define new types of Laplacians and of Schrödinger operators on infinite graphs. We find that under some conditions these operators are essentially self-adjoint. We present some recent results obtained in joint work with Yves colin de Verdière and Françoise Truc.





### Carthaginians on the Source of their Wealth: Olive Oil and Trade Ibn Adhari al-Marrakushi (b. 1295)

البيان المُغْرِب في اختصار أخبار ملوك الأندلس والمغرب

ابن عذاري ( ولد سنة 1295 م)

ولما انهزمت جيوش جرجير سار عبد الله بن أبي سرح حتى نزل باب مدينته العظمى قرطاجنة، فحصر ها من كان معه من المسلمين حصارا شديداحتى فتحت فأصاب فيها من السبي والأموال ما لم يحط به الوصف وكان أكثر أموالهم الذهب والفضة فكانت توضع بين أيديه أكوام الفضة والذهب لأنه افترع أفريقية مبكرا فعجب هو والمسلمون من كثرة ذلك فقال للأفارقة: (من أين لكم هذا؟) فجعل الرجل منهم يلتمس شيئاً من الأرض حتى بنواة زيتون، فقال:(من هذا أصبنا الأموال لأن أهل البحر والجزر ليس لهم زيت فكانوا يمتارونه من هنا).

When the armies of the [Byzantine Patrician] Gregorious were defeated [in 647 AD], Abdallah ibn Abi Sarh rode [with his pride from Sbeitla] until he reached the gate of Gregorious' Grand City Cartagena. He laid a fierce siege to it with whatever was left of his Muslim army. When the city was finally conquered, Abdallah collected of spoils and money what no man can describe. Most of the Carthaginian money was either gold or silver. They were heaped in piles before him for he conquered Africa before it could be defended. He and his men were in awe from the abundance of wealth. Thus he asked the Africans: "Where did you collect all of this?" Said a man who was fetching something from the ground and found an olive pit: "It is from this that we collected all these moneys, for the peoples of these islands around us did not have oil and used to buy it from us".

### ACADEMIC SPONSORS









### INTENSIVE COURSES AT TUNIS SCIENCE CITY

### COURSE I:

## Isoperimetric Inequalities for Eigenvalues of the Laplacian

### Rafael Benguria, Depto. Física, PUC, Santiago de Chile

This minicourse will consist of four fifty minute lectures on isoperimetric inequalities for eigenvalues of the Laplacian. The summary of the contents of the lectures will be as follows.

Lecture 1: I will start the minicourse with a review of Mark Kac's problem: "Can one hear the shape of a drum?" starting with a brief review of Kac's classical paper of 1966 and the history around that question up to today.

**Lecture 2:** This lecture will consist of two parts: in the first part of the lecture I will review the history of isoperimetric inequalities in mathematical physics. In the second part I will give a short course on symmetric decreasing rearrangements, which are a fundamental tool in the proof of isoperimetric inequalities for the eigenvalues of the Laplacian.

Lecture 3: In this lecture I will start with the proof of the Rayleigh–Faber–Krahn inequality for the lowest eigenvalue of the Dirichlet Laplacian and some of its extensions. I will also give the proof of the Szegö–Weinberger inequality for the first nontrivial Neumann eigenvalue. Then, I will give a proof of the Payne–Pólya–Weinberger inequality. I will end this lecture with a short review of other isoperimetric inequalities for eignevalues and a list of some open problems.

Lecture 4: In the final lecture I will discuss isoperimetric inequalities for plates, including some open problems.

**References:** During the course of this set of lectures I will give many references on the subject. For the convenience of the participants of this mini–course, I refer to the two recent review articles:

 M. S. Ashbaugh, and R. D. Benguria, *Isoperimetric Inequalities for Eigenvalues of the Laplacian*, in Spectral Theory and Mathematical Physics: A Festschrift in Honor of Barry Simon's 60th Birthday, Proceedings of Symposia in Pure Mathematics Vol. 76, Part 1, F. Gesztesy, P. Deift, C. Galvez, P. Perry, and W. Schlag, eds., Amer. Math. Soc., Providence, RI, pp. 105–139 (2007).

[2] R. D. Benguria and H. Linde, *Isoperimetric Inequalities for the Eigenvalues of the Laplace Operator*, in **Topic in Spectral Theory and Quantum Mechanics**, 4th Summer School in Analysis and Mathematical Physics, Carlos Villegas–Blas (Editor), Contemporary Mathematics **476**, 1–40 (2008).

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### COURSE II:

### Laplacian on Riemannian Manifolds

### Bruno Colbois, Université de Neuchâtel, Suisse

Lecture 1: I will introduce the laplacian on a Riemannian manifold (M, g) and give some basic properties. If the manifold M is compact, the spectrum of the laplacian is discrete and is denoted by  $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \ldots$ . Then, we can see the k-th eigenvalue  $\lambda_k$  as a functional on the set of volume 1 metric on M. I will discuss to what extent this functional may take values arbitrarily close to 0 (example of the Cheeger dumbbell) or arbitrarily large: in other words, on a given manifold, is it possible to construct metrics of volume 1 and arbitrarily large or arbitrarily small eigenvalues?

I will finish by introducing the Cheeger constant and I will give and explain (without the detail of the proofs) the Cheeger and the Buser inequalities about  $\lambda_1$ .

**Prerequisites:** Basic notions of Riemannian geometry as in [Be] Chapter 2 or the chapter "Basic Riemannian Geometry" in [DS].

Lecture 2: I will discuss the influence of the curvature on the spectrum. I will first survey some classical results like:

- Li-Yau estimate for compact manifolds with Ricci curvature bounded below
- The upper bound theorem of Cheng (see [Ch] a reference for this part).

I will then focus on the case of negatively curved manifolds: what knowledge about the spectrum bring the fact to be negatively curved? A reference for this part may be [Bu]

**Lecture 3:** Estimate on the conformal class. A classical question is to find upper bound for the eigenvalues for the volume 1 metric g on the conformal class of a given metric  $g_0$ . I will present the classical result of Korevaar as exposed in the article [GNY], chapter 3 and 4. In particular, the method of chapter 3 of [GNY] is interesting by itself and has a lot of applications.

Lecture 4: I will present the case of a submanifolds of the euclidean space. I will present some sharp inequalities for the first nonzero eigenvalue  $\lambda_1$  and discuss the equality case. I will finish by explaining a method developed with D. Maerten and which allows to extend some of these inequalities to the other eigenvalues.

Of course, this program is subject to certain modifications.

#### References

[Be] Bérard, P.: Spectral Geometry: Direct and Inverse Problems, Lecture Notes in Mathematics, 1207 (1986).

[Bu] Buser, P.: On Cheeger's inequality  $\lambda_1 \ge h^2/4$ , in Proc. Symposia in Pure Mathematics, Vol. 36 (1980) 29-77.

[CE] Colbois B., El Soufi A.: Extremal Eigenvalues of the Laplacian in a Conformal Class of Metrics: The "Conformal Spectrum", Annals of Global Analysis and Geometry 24, (2003) 337-349.

[Ch] Chavel, I.: Eigenvalues in Riemannian Geometry, Ac. Press, 1984.

[DV] B. Davis, Y. Safarov (editors), Spectral Theory and Geometry, LMS Lecture Note Serie 273, Cambridge Univ. Press, 1999.

[GNY] A. Grigor'yan; Y. Netrusov; S.T. Yau; Eigenvalues of elliptic operators and geometric applications. Surveys in differential geometry. Vol. IX, 147217, Student who want to have a first view of the subject may look at the article of Chavel "The Laplacian on a Riemannian Manifold" of the reference [DV].

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### COURSE III:

### Isoperimetric inequalities in negatively curved spaces

### Olivier Druet, École Normale Supérieure de Lyon, France

**Lecture 1:** We will review some proofs of the isoperimetric inequality in the Euclidean space and recall the links between isoperimetric inequalities and Sobolev inequalities.

Lecture 2-3-4: We will study the isoperimetric inequality in non-positively curved, complete, simply connected Riemannian manifolds. A well-known conjecture, which is still open in general, says that, on such manifolds, the Euclidean isoperimetric inequality should hold. We will try to discuss the three proofs of this conjecture in dimensions 2 (by Weil), 3 (by Kleiner) and 4 (by Croke).

These proofs are really different one from the other and this will give an overview of some tools that can be used to attack this problem. At last, if there is time, we will discuss a local version of this inequality and its consequence on the isoperimetric profile and Faber-Krahn profile of a Riemannian manifold.

# Tunis Science City

The Tunis Science City is an institution in charge of disseminating scientific culture over the whole country, to every category of the population, and particularly to young people.

It is composed of several halls, each of them is dedicated to a specific scientific theme. They are as follows:

- A Planetarium, seating 120 very comfortably and allowing the projection of more than 4,000 stars on a hemispherical screen,
- The Earth and the Universe pavilion, where hands-on displays help visitors to get a better grasp of astronomical concepts,
- Life and Man on Earth pavilion, offering an evolutionary view of life on Earth from the beginning to the present,
- Explora pavilion including 96 props dealing with physics, mathematics, biology, chemistry,
- Temporary exhibitions pavilion dedicated to house, for a limited period, thematic exhibitions.
- Water pavilion dealing with water, its components and different states
- A Multi media library divided into two sections; the children media section and the public media section,
- A Information Access Centre giving a free access to Information and Communication Technologies.
- A Conference Centre fitted with a high-tech projection equipment and visio conference system.

There are also other key exhibits illustrating the educational mission of the Tunis Science City on display in the open air:

- Archimedes' Screw
- The Crown of the main entrance,
- Foucault's pendulum,
- Clepsydra or water clock,
- The Noria or paddle-wheel,
- Sundial.

The Tunis Science City also disseminates science to the interior of the country to get science close to every citizen wherever he may be, thanks to a bus baptized Al Katira carrying on its board travelling exhibitions and a mini-planetarium.





### ORGANIZERS



Mark Ashbaugh University of Missouri, Columbia, USA ashbaughm@missouri.edu

Mark Ashbaugh earned his undergraduate degree at Grinnell College in Grinnell, Iowa, where he majored in Mathematics and Physics. He received his Ph.D. in Mathematical Physics at Princeton University in 1980, completing his dissertation under the direction of Barry Simon. He works on problems in differential equations and quantum theory, particularly on eigenvalue problems for differential operators and other topics in spectral theory. He is currently Professor of Mathematics at the University of Missouri in Columbia, Missouri. He has visited in Chile many times, and was a Fulbright Scholar at the P. Universidad Católica de Chile in Santiago during the latter part of 2009. He was elected a Foreign Corresponding Member of the Academia Chilena de Ciencias in November, 2009.

#### Title: The Buckling Problem and the Krein Laplacian

Abstract: Recent developments on the buckling problem and the Krein Laplacian in which the author has been involved will be discussed, including connections between these two problems, analysis of their spectral asymptotics, and inequalities for their eigenvalues. In particular, we note that the buckling problem is intimately related to the Krein Laplacian, and that, in fact, there is a unitary equivalence between the two problems if one considers the Krein Laplacian on the space orthogonal to its kernel. Old conjectures concerning the eigenvalues of the buckling problem will also be discussed, including the Pólya-Szegő conjecture for the first eigenvalue (which would be the Faber-Krahn result for this problem) and Payne's conjecture comparing the buckling eigenvalues to those of the Dirichlet Laplacian on the same domain.

Much of the recent work presented in the talk represents joint work with Fritz Gesztesy, Marius Mitrea, Roman Shterenberg, and/or Gerald Teschl.





Lennie Friedlander University of Arizona USA friedlan@math.arizona.edu math.arizona.edu/~friedlan

Leonid (Lennie) Friedlander started doing research in spectral theory of non-selfadjoint operators under the direction of Mikhail S. Agranovich. After having moved from the Soviet Union to the United States, he went to graduate school in MIT; he got his PhD in 1989 (thesis "Determinants of Elliptic Operators"; advisor Victor Guillemin.) From 1989 to 1991 he was an Adjunct Assistant Professor in UCLA, and from 1991 he is working in the University of Arizona, currently as Professor of Mathematics.

Main interests: spectral asymptotics, eigenvalue estimates, periodic problems, determinants of elliptic operators.

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Habib Bouchriha Faculté des Sciences de Tunis Tunisia habib.bouchriha@fst.rnu.tn

Habib Bouchriha is Professor of Physics at the Faculty of Sciences of Tunis and Director of the Centre de Publication Universitaire. Born in 1947, Bouchriha received his Baccalauréat in 1965 and Licence Es

Sciences Physiques in 1969. He received his Doctorat 3ème cycle Physique (1972) and Doctorat d'état Es Sciences Physiques (1978) from Paris. Bouchriha was Maître Assistant at Université Paris 7 (1976-1978), Maître de Conférences at the Faculté de Sciences de Tunis (1978-1982), and Full Professor since 1982. Bouchriha authored 109 scientific articles published in index international reviews, 10 polycopied courses, and 8 pedagogical books. His 2-volume book "Mécanique" (with Zeineb Ben Ahmed and Dhouha Gamra) won the prix Roberval des oeuvres pour comprendre la technologie.

His research centers around the Physics of Condensed Matters, Polymers, Photonics, Quantum Mechanics, Optoelectronics, Magnetic Resonance, and Nanosciences. He holds the Médaille du Mérite Éducatif Tunisien (1994), the Rammal Award of the Société Française de Physique et de la Fondation de l'Ecole Normale Supérieure de Paris, and is Commandeur de l'ordre du mérite national au titre du secteur de l'éducation et de l'enseignement (2009). Founding President of the Tunisian Physical Society (1980-1984), President of the Arab Union of Physicists and Mathematicians (1979-1982), former member of the Scientific Council of the Maghreb Academy of Sciences (1993-1995), former member of the Scientific Council of I.N.S.A.T, ENS, EPT, INRAP, Cité des Sciences, and Président of the National Council for LMD Reform in Tunisia since 2005. Professor Bouchriha directed 60 undergraduate and masters theses, 35 doctoral dissertations, and 22 thèses détat and habilitations.





Evans Harrell Georgia Institute of Technology USA harrell@math.gatech.edu people.math.gatech.edu/~harrell

Evans Harrell is a Professor of Mathematics at Georgia Tech. He studied at Stanford and Princeton, where he wrote a dissertation with Barry Simon. Before coming to Georgia Tech from Johns Hopkins in the 1980s he had a series of postdocs and teaching positions in the US and Europe, and he has held several visiting positions in France since then. While at Georgia Tech he has received a Sloan Fellowship and an Eichholz Faculty Teaching Award, and he is a Fellow of the American Association for the Advancement of Science. Before his appointment as Associate Dean of Sciences in late 2005, he managed the graduate programs of the School of Mathematics.

Harrell works on the Schrödinger equation, which describes nonrelativistic matter on the atomic scale, and on similar differential equations. His research has focused primarily on semiclassical quantum mechanics and on the effects of geometry on the eigenvalues of differential equations. He has advised eight PhD dissertations and two MS theses, and has written numerous articles with at least thirty-five coauthors.

#### Title: Eleven spectral properties of the sphere

Abstract: Ninety years ago, Hilbert gave a lecture at Göttingen on isoperimetric theorems, entitled "Elf Eigenschaften der Kugel," or "Eleven Properties of the Sphere," a version of which later appeared as a chapter in his charming book with Cohn-Vossen, *Anschauliche Geometrie*, translated into English as *Geometry and the Imagination*. In this lecture I shall describe eleven ways in which one can "hear" that a region is round, i.e., a selection of facts about the eigenvalues of the Laplace operator or of some special Schrödinger operator which can be true if and only if the set upon which it acts is round. I shall begin by recalling some classical theorems, and then offer some perspectives on the interplay of analysis, geometry, and algebra underpinning these spectral isoperimetric theorems and others I have obtained with numerous coauthors.





Alumnus of the Lycée Khaznadar du Bardo, Lotfi Hermi left Tunisia in 1986 on a national merit scholarship to study Mechanical Engineering at the University of Missouri–Columbia, which he completed in 1990. He has an M.S. in Applied Mathematics from Ohio State University (1992) and a Ph.D. in Mathematics from University of Missouri–Columbia (1999). His dissertation, On the Spectrum of the Dirichlet Laplacian and Other Elliptic Operators, was supervised by Professor Mark Ashbaugh. After a year at Marshall University, he moved to University of Arizona where he is now Assistant Professor of Mathematics. His academic carrier includes a visit to Georgia Tech (Fall 2006) where universal bounds for eigenvalues in tune with the Weyl asymptotics were explored with Professor Evans Harrell.

His research interests center around various aspects of spectral analysis, specifically dealing with geometric and universal bounds of eigenvalues, spectral isoperimetric inequalities, semiclassical analysis, and their applications to pattern recognition. Together with Deb Hughes Hallett and Bill McCallum he developed materials exploring aspects of voting theory (math.arizona.edu/~voting-theory), and has co-organized with them and M. Ben Rhouma the 2007 "Learning Technologies and Mathematics Middle East Conference" which was held at Sultan Qaboos University, Muscat, Oman.



Najoua Gamara Faculté des Sciences de Tunis Tunisia ngamara7@gmail.com

Najoua Gamara was educated at Paris VII and Université de Savoie. She wrote her M.S. thesis working with Paul Gérardin *On Einstein manifolds of dimension* 4 in 1982, and her doctorat d'état from Savoie working with Pierre Bérard in 1984 on *Symmetrization of elliptic equations*. She earned her Habilitation in 2002 by resolving the Yamabe CR-conjecture working with Abbas Bahri. She has been with the Faculté des Sciences de Tunis since 1984.

Gamara was a national athlete (1973-1990), champion of Tunisia and Arab countries in the long-jump, vicepresident of the Tunisian Athletic Federation (1994-1999) in charge of the promotion of women athletics, and member of the African Athletic Confederation since 1997 in charge of the North Africa area. She was nominated in 2001 by the Minister of Youth and Sports to the Executive Board of the Tunisian Athletic Confederation, and councelor of the President of the Tunisian Athletic Confederation since 2006. Executive member of the Tunisian Mathematical Society, her interests include symmetrization, CR and Heisenberg geometries, the Yamabe problem, and the interplay between Riemannian geometry and Morse theory.



Habiba Guemri-Boukraa Faculty of Sciences of Tunis Tunisia habiba.guemri@gmail.com

Habiba Guemri just completed her PhD dissertation under the supervision of Professor Najoua Gamara entitled "Résultats d'existence pour des équations semi-lineaires critiques sur des domaines du groupe de Heisenberg". She is currently Assistant at the Institut Supérieur d Informatique de Medenine, Universite de Gabès. Her research interests includes geometric and spectral aspects of Heisenberg geometry and the related extremal questions.

Title: Existence results for critical semi-linear equations on Heisenberg group domains

Abstract: Following the work of G. Citti and F. Uguzzoni who studied Yamabe type problems on Heisenberg group domains, we consider the critical semi-linear equation on a domain of the Heisenberg group  $\mathbb{H}^1$ :

$$(P) \begin{cases} -\Delta_H u = K u^3 & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{on} \partial \Omega \end{cases}$$

where  $\Delta_H$  is the sublaplacian on  $\mathbb{H}^1$  and K is a  $C^3$  positive function defined on  $\Omega$ . Using a version of the Morse Lemma at infinity, we give necessary conditions on K to insure the existence of solutions for (P).

Abderrazek Karoui Faculté des Sciences de Bizerte, Tunisia Abderrazek.Karoui@fsb.rnu.tn

Abderrazek Karoui received the BSc in mathematics from the faculty of Sciences of Tunis in 1990, the MSc and Ph.D. degrees from the University of Ottawa, Canada in 1992 and 1995, respectively. In 2004, he has received the Habilitation degree in Mathematics from the faculty of Sciences of Tunis, Tunisia. Dr. Karoui is presently a full Professor of Mathematics at the faculty of Sciences of Bizerte, University of Carthage, Tunisia. His main research area and interests include Applied harmonic analysis (Fourier Analysis, Time Frequency Analysis Tools such as Wavelets, Prolate Spheroidal wave Functions), Integral Equations, Approximations and expansions, spectral methods based on orthogonal polynomials and special functions.

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M. Néjib Lazhari Faculté des Sciences de Tunis Tunisia mednejib.lazhari@mes.rnu.tn

Professor Mohamed Néjib Lazhari is Director of Programs and Scientific Partnerships at the Tunisian Ministry of Higher Education and Scientific Research.

#### 



Fethi Mahmoudi Faculté des Sciences de Tunis Tunisia fethi.mah@gmail.com

Fethi Mahmoudi received his Ph.D. in Mathematics in 2005, from the University of Paris 12 (France), under the supervision of Professor Frank Pacard. After 2 years of postdoctoral work with Professor Andrea Malchiodi at the International School of advanced Studies in Trieste, Italy, he joined the Faculty of Sciences of Tunis in 2007 where he obtained an associate professor position. His main interests are in Geometric Analysis, Nonlinear Partial Differential Equations, Differential Geometry, Riemannian Geometry. With a particular focus on problems related to concentration phenomena for (geometric) nonlinear elliptic equations: Constant r-mean curvature hypersurfaces in Riemannian manifolds, Singularly Perturbed Problems, Free boundary problems, Nonlinear Schrödinger (NLS) equations, and other related problems.

Title: Hypersurfaces with large constant mean curvature and free boundary

Abstract: Given a domain  $\Omega$  of  $\mathbb{R}^{m+1}$  and a k-dimensional non-degenerate minimal submanifold K of  $\partial\Omega$  with  $1 \leq k \leq m-1$ , we prove the existence of a family of embedded constant mean curvature hypersurfaces in  $\Omega$  which as their mean curvature tends to infinity concentrate along K and intersecting  $\partial\Omega$  at a given constant angle  $\gamma$  along their boundaries.

This is joint work with Mouhamed Moustapha Fall.





Mohamed Sifi Faculté des Sciences de Tunis Tunisia mohamed.sifi@fst.rnu.tn www.ipeit.rnu.tn

Mohamed Sifi obtained his Docteur 3ème cycle under the supervision of A. Achour and K. Trimèche (1992), and Docteur d'Etat es-Sciences Mathématiques working with K. Trimèche (1996). He joined the Faculty of Sciences of Tunis where he was Maître Assistant (1992), then after a brief period as a Maître de Conférences at the École Supérieure des Sciences et Technique, full Professor in 2001. He was Head of the Math. Department at the Fac. des Sciences (2008-2009). Sifi is Head of the Preparatory Institute of Engineering Studies of Tunis since 2009. He served as a member of the Executive Committee of the Tunisian Mathematical Society since 2003 and is currently its Vice-President. Sifi held visiting positions in France at Univ. of Orléans (2008) and Univ. of Nancy (2009). He is Associate Editor of Adv. in Pure and Appl. Math. and author of more than 27 research papers. His research interests are: Harmonic Analysis associated with PDE's; special functions associated with root systems: rational (Dunkl) and trigonomoetric (Opdam, Cherednick) settings.

Khalifa Trimèche Société Mathématique de Tunisie Tunisia Khlifa.Trimeche@fst.rnu.tn www.tms.rnu.tn

Professor Khalifa Trimèche received his Doctorat 3ème cycle (1974), and Doctorat d'Etat es-Sciences Mathématiques (1980), both under the supervision of J. Faraut. Maître Assistant (1976), and Maître de Conférences (1980) at Faculty of Sciences of Tunis, he became full Professor in 1985. Trimèche was Head of the Mathematics Department there (1985-1990). He has served as Vice President of the Tunisian Mathematical Society (1993-1995) and has been President of this society since 2003. Professor Trimèche has five published books and 115 articles. He directed 7 theses d'etats, 7 theses de 3ème cycle, 7 doctorats d'etats, and 4 habilitations. He was a visiting professor in several universities in France, India, and Morocco.

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Hamdi Zorgati Faculté des Sciences de Tunis Tunisia hamdizorgati@yahoo.fr www.ann.jussieu.fr/~zorgati

Hamdi Zorgati completed a University degree in mathematics at the Faculté des Sciences de Tunis in 2000, then the Master in Numerical analysis in University Pierre et Marie Curie, Paris 6 in 2001. His PhD was with Professor Hervé Le Dret at the Laboratory Jacques-Louis Lions of the same university which he defended in December 2004. The subject of the thesis dealt with dimension reduction for thin films using tools of calculus of variations and principally  $\Gamma$ -convergence. After working for two years at Université de Paris Dauphine he moved to the Faculté des sciences de Tunis, where he has been since 2006. His principal fields of research are the Calculus of Variations, Non-Linear Analysis, Numerical Analysis and Solid Mechanics.

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الروض المعطار في خبر الأقطار The Book of the Fragrant Garden للجغرافي الأندلسي (1461 م) محمد بن عبد المنعم الحِميَري وتونس من أشرف مدن إفريقية وأطيبها ثمرة وأنفسها فاكهة،

وسميت تونس لأن المسلمين كانوا لما فتحوا إفريقية ينزلون بازاء صومعة ترشيش- راهب كان هناك- ويأنسون بصوت الراهب فيقولون: هذه الصومعة تؤنس، فلزمها هذا الاسم.

**†** unis is among the most noble cities of Africa, with the sweetest of fruits and most precious of nuts. It was called Tunis by the Muslims for when they conquered it they set base close to the tower of Tarshish-a Christian monk who worshipped there. They found his voice soothing-and said "this is a soothing tower". Thus the town came to be known as the "city of soothing companionship", Tunis.

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### NAMES AND THEIR HISTORIES A Handbook of

### HISTORICAL GEOGRAPHY

#### and

### TOPOGRAPHICAL NOMENCLATURE

### 1898

**Tunis**, in North Africa, preserves the ancient name of a fortified town called *Tunês* by Polybius and Diodorus, and *Tunis* by Strabo, dedicated, it is supposed, to the Phœnician goddess Tanith, identified by the Greeks with Artemis, to whom several inscriptions have been found at Carthage.

### SPONSORS







### CONTRIBUTED TALKS



Denis Abramov Kazan State University Russia quaerix@gmail.com

Denis Abramov was born in Naberezhnye Chelny, The Russian Federation. He graduated from Kazan State University in 2007. He is a post-graduate student. His science advisor is Professor Farit G. Avkhadiev. His scientific interests include isoperimetric inequalities in mathematical physics (particularly, inequalities related to torsional rigidity), Hardy-type inequalities and approximate computations.

## Title: Estimates for the equivalence constant of the torsional rigidity coefficient and the Euclidean moment of inertia

Abstract: We consider the torsional rigidity problem for simply connected domains. The field of research is equivalence relations between the torsional rigidity coefficient and Euclidean moment of inertia introduced by F. G. Avkhadiev. For this analysis we use approximate computation of the torsional rigidity coefficient. Besides, the approach for distance function determination is defined, a numerical method for solving of the Euclidean moment of inertia is proposed. Using the developed program, an estimate for the equivalence constant of given functionals has been improved. We also examine the behavior of the equivalence constant on certain domains classes and produce the domains, which may tend to the position of "extremal domains" for this equivalence relation.

This research is supported by Federal Target Program "Scientific and Scientific-Pedagogical Personal of Innovation Russia" (contract N 02.740.11.0193).



Hassan Ait Haddou Institut de Recherche en Informatique de Toulouse France haithaddou@yahoo.com

Hassan Ait Haddou is currently a Postdoctoral fellow at IRIT, Toulouse, France. His 2005 PhD dissertation "On the Lichnerowicz Cohomology" was supervised by Augustin Banyaga from PennState University, and was received in Italy.

#### Title: Foliation and twisted cohomology

Abstract: Let F be a *n*-dimensional foliation on a closed manifold M. The aim of this work is to introduce and give some properties of the Lichnerowicz basic cohomology which is an important generalization of the basic cohomology. In [1, 2] we have showed the Leray-Hirsch theorem for basic and vertical forms. In this note we will generalize this theorem for all differential forms. We will also prove that many properties of the usual basic cohomology still have their analogues within the Lichnerowicz basic cohomology. We will also compute this new cohomology for some foliations. In particular, we will show that the Lichnerowicz basic cohomology of 0 degree of any connected manifold is trivial which is not the case for the basic cohomology. The Gysin sequences [4] are used to give the relationship between the Lichnerowicz cohomology [3] and the Lichnerowicz basic ones. We will use this sequence to give some properties of Lichnerowicz basic cohomology for given foliation.

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Amine Amri Faculté des Sciences de Tunis Tunisia amineamri.fst@gmail.com

Amine Amri obtained his Masters degree from Université de Montpellier 2 in France. He spent a research internship at Université d'Avignon and since March 2007 has been a PhD student at the Faculty of Sciences of Tunis working under the supervision of Professor Najoua Gamara. He is currently an academic assistant at the Institute of Computer Sciences and Multimedia of Gabès.

His research interests are in Cauchy-Riemann Geometry, Critical Points at Infinity, Set-Valued Analysis and Linear Complementarity Problems.

#### Title: Spectral analysis of coupled linear complementarity problems

Abstract: This talk deals with the so-called cone-constrained bivariate eigenvalue problem. The equilibrium model under consideration is a system of linear complementarity problems

$$\begin{cases} P \ni x \perp (Ax + By - \lambda x) \in P \\ Q \ni y \perp (Cx + Dy - \mu y) \in Q \end{cases}$$

involving two closed convex cones and their corresponding duals. We study the set of pairs  $(\lambda, \mu) \in \mathbb{R}^2$  for which this system has a "nontrivial" solution  $(x, y) \in \mathbb{R}^{n+m}$ . We discuss also the link between the cone-constrained version and the unconstrained one.

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Farit Avkhadiev Kazan State University Russia favhadiev@ksu.ru

Farit Gabidinovich Avkhadiev was born in 1947. He has been at the Kazan State University since 1969, as a researcher 1969-1994, and professor (1994-present). He speaks French since he spent three years at Annaba University (1977-1980).

#### Title: Hardy type inequalities with sharp constants

Abstract: We consider Hardy type inequalities in the form of H. Brezis and M. Marcus in spase domains with two unknown constants. Our main aim is to fine sharp values of the constants when the domains satisfy certain isoperimetric restrictions. We will present several new Hardy type inequalities with two sharp constants for convex and some classes of nonconvex domains.





Sidi Mohamed Bahri Faculté des Sciences de Mostaganem, Algeria bahrisidimohamed@yahoo.fr

Sidi Mohamed Bahri was born in Nedroma, Algeria. In 1990 he got his Diploma of Higher Education in Mathematics from Oran University. He got his Masters in Mathematics from the Ecole Normale Supérieur of Mostaganem in 1995. His advisor was E. L. Alexandrov (Saratov University). He received his Doctorat Es Sciences in Mathematics from Mostaganem University in 2007. He is currently preparing his accreditation to supervise research, scheduled for this year. His areas of research are: Spectral study of Carleman integral operators, Description in terms of analytic functions of sets' resolvents and generalized spectral functions, Perturbation theory, Applications to Stochastic processes and estimation of a random variable.

#### Title: Spectral Aspects of a Certain Class of Carleman Operators

Abstract: In this talk, I will introduce an operation that gives a different form to the Carleman integral operator, namely

$$A_{\alpha}f = \alpha \circ f = \sum_{n} \langle \alpha, \psi_n \rangle \langle f, \psi_n \rangle \psi_n, \quad \alpha = \sum_{n} \alpha_n \psi_n,$$

 $\{\psi_n\}$  an orthonormal sequence in  $L_2$ .

With (1), we can easily study the Carleman operators disturbed by a position operator  $U\psi_n = \psi_{j(n)} = \psi_n^v$ (j an involution) :

$$A = A_{\alpha} + A_{\beta}U = \alpha \circ f + \beta \circ \overset{v}{f}, \qquad f \in D(A), \qquad (1)$$

where D(A) is the domain defined by

$$\left\{f \in \mathcal{L}_{\psi}: \left(\sum_{n} \left|\alpha_{n}\left(f, \psi_{n}\right)\right|^{2} + \sum_{n} \left|\beta_{n}\left(f, \psi_{n}\right)\right|^{2}\right) < \infty\right\}.$$



Lamia Bel Kenani Toukabri Faculty of Sciences of Tunis Tunisia lamiatoukabri@yahoo.fr

Lamia Bel Kenani Toukabri is an Academic Assistant of Mathematics at the Institut préparatoire aux études d'ingénieurs de Tunis (IPEIT). She is about to defend her Ph.D. thesis in PDE's, entitled *"Existence et Regularité de solution aux équations de Monge-Ampère dans des domaines non bornés"* working with Professor Saoussen Kallel-Jallouli.

Title: Geometric interpretations and utilities related to the Monge-Ampère equation

Abstract: We consider the Monge-Ampère equation

$$\det(D^2 u) = \Psi(x, u, D u) \text{ in } \mathbb{R}^n, n \ge 3,$$

where  $\Psi$  is a positive function in  $C^2 (\mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^n)$ . We prove the existence of convex solutions, provided there exist a subsolution of the form  $\underline{u} = a |x|^2$  and a superharmonic bounded positive function  $\phi$  satisfying:  $\Psi > (2a + \Delta \phi/n)^n$ .



Ali Ben Amor Faculté des Sciences de Gafsa, Tunisia ali.benamor@ipeit.rnu.tn

Ali Ben Amor received his PhD from the Faculty of Sciences of Tunis in 2001. He held postdoctoral positions at University of Bielefeld, Germany (2001-2002) and University of Osnabrück, Germany (2003-2005), and was Associate Professor at the University of Osnabrück (2005-2006) before returning to Tunisia. Maître Assistant at IPEI-Tunis (2006-2008), he's been Associate Professor at Faculty of Sciences of Gafsa since 2009. His research interests are: Spectral problems related to perturbed Dirichlet operators; inequalities in the scope of Dirichlet spaces.

#### Title: Estimations of the ground state for for some classes of positivity preserving operators

Abstract: We establish a comparison result for the ground state of some classes of positivity preserving operators. The considered operators are negatively perturbed strongly local Dirichlet operators by means of measures. The result will deserve to obtain heat kernel and Green function estimates for the related operators.



Ines Ben Omrane Faculté des Sciences de Tunis Tunisia benomraneines@gmail.com

Ines Ben Omrane has just defended her PhD dissertation entitled "Etudes de quelques équations aux dérivées partielles non-linéaires issues de la géométrie et de la physique" under the supervision of Professor Sami Baraket. Her research interests include compensation phenomena (Wente problem) and existence of branches of solutions having singular limits for some nonlinear problems.

 $\label{eq:constraint} \textit{Title: Singular limits for 2-dimensional elliptic problem involving exponential nonlinearity with nonlinear gradient term$ 

Abstract: Given a bounded open regular set  $\Omega \subset \mathbb{R}^2$  and  $x_1, x_2, \dots, x_m \in \Omega$ , we give a sufficient condition for the problem

 $-\operatorname{div}(a(u)\nabla u) = \rho^2 f(u)$ 

to have a positive weak solution u in  $\Omega$  with u = 0 on  $\partial \Omega$ , which is singular at each  $x_i$  as the parameter  $\rho$  tends to 0 and under suitable assumptions on exponential functions a(u) and f(u).

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Mohamed Ben Rhouma Sultan Qaboos University Oman rhouma@squ.edu.om

Mohamed B. H. Rhouma received the degree of Ingenieur Principal in industrial engineering from the Ecole Nationale d'Ingenieurs de Tunis, Tunisia in 1989 and both his Msc and Ph.D in mathematics from the University of Missouri Columbia in 1995 and 1999 respectively. He did his postdoctoral training at the Center for Dynamical Systems and Nonlinear Studies at Georgia Institute of Technology in Atlanta. He is currently an assistant professor of Mathematics at Sultan Qaboos University in Oman. Dr Rhouma's main area of research is dynamical systems and their applications in pattern recognition.

#### Title: We may hear the shape of a drum

Abstract: In this talk, we will present the results of an experimental study on the use of the eigenvalues of the Laplacian and Bi-Laplacian operators to extract feature vectors that are able to yield a 90%+ rate of recognition on real shapes data sets. (joint work with L. Hermi and M. A. Khabou)



Chiara Bianchini Université Henri Poincaré, Nancy 1, France chiara.bianchini@math.unifi.it

Chiara Bianchini is an ANR CNRS post-doctoral fellow at the Institut Elie Cartan in Nancy, France.

- Bachelor degree on "Planar Isoperimetric Problems" (Università degli studi di Firenze), 2004;
- Master Thesis on "Rogers & Shephard type inequalities for the p-difference body of a convex body" (Università degli studi di Firenze), 2006;
- PhD Thesis on "Convex Aspects of Elliptic problems" (Università degli studi di Firenze), supervisors: Prof. A. Colesanti and Dott. P. Salani, 2010.

Her research interests focus on geometrical and functional inequalities, geometric properties of solution to elliptic PDE, isoperimetric functions, Brunn-Minkowski theory.

#### Title: Isoperimetric bounds for product probability measures

Abstract: Given a probability measure  $\tau$  we define its *Isoperimetric function* for  $t \in [0,1]$ , as  $I_{\tau}(t) = \inf\{\tau^+(\partial A) : \tau(A) = t\}$ , where  $\tau^+(A)$  indicates the *boundary measure* of a sufficiently smooth set A. It is known from [BCR] that, under suitable assumptions on  $\tau$ , an isoperimetric inequality of the following type holds:  $I_{\tau^N}(t) \ge c_{\tau} I_{\tau}$ , where  $\tau^N$  is the product measure of  $\tau$ .

Since by definition it always holds  $I_{\tau^N}(t) \leq I_{\tau}$ , these give a two-sided bound for the isoperimetric functions. We analyse these inequalities for product measures  $\mu^N$ , where  $\mu$  is the logistic measure

$$d\mu(x) = \frac{e^x}{(1+e^x)^2} \, dx,$$

 $x \in \mathbb{R}$  finding a value for the constant  $c_{\mu}$  which bounds more accurately the range of  $I_{\mu^N}$ , compared with what is known in the exponential case due to [BH].

Moreover, we gave a characterization of stationary and stable half spaces for  $\mu$ , using results in [RCBM].

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Hakima Bouhadjera Badji-Mokhtar University of Annaba, Algeria b\_hakima2000@yahoo.fr

Hakima Bouhadjera is a Mathematics PhD student at the Faculté des Sciences, Université Badji Mokhtar de Annaba, Algeria, originally from El Tarf. She is Maître Assistant, classe "A", Attachée de recherche au Labo. de Math. Appliquées. She received her Masters in specializing in Functional Analysis, Compatible Applications and Fixed Point Theorems in 20003 and her D.E.A. in Dynamical Systems in 2001, both from University of Annaba, Algeria. Her D.E.S., received in 1993, is in Numerical Analysis and Operational Research.

Title: General common fixed point theorems for occasionally weakly biased mappings in different spaces

Abstract: We introduce a very general notion of compatible and biased mappings, and we prove some common fixed point theorems for four mappings in different spaces. These theorems are generalizations of some results existing in the field of commuting, compatible and biased mappings.

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Salah Boulaaras Institut Hydrometeorologique de Formation et de Recherche, Oran, Algeria saleh\_boulaares@yahoo.fr

Salah Boulaaras received his B.S. in Mathematics in 2006 and M.S in Stochastic Calculus both from Université de Badji Mokhtar of Annaba, Algeria. He is currently working on a Ph.D. at Université de Badji Mokhtar specializing in Numerical Analysis and is Maître Assistant at Institut the Hydrometeorologique de Formation et de Recherche, Oran, Algeria.

#### Title: Convergence of the method Schwarz for a impulse control problem

Abstract: In this work we provide an uniform error estimate using an overlapping Schwarz method on nonmatching grids for quasi-variational inequalities with the obstacle of impulse control, this our study is is based on the M. Haiour's study [1] and M. Boulbrachene's study [2]; The main idea of this method consists in decomposing the domain in two subdomains. We provide that the discretisation on every subdomain converges in uniform norm and we give a result of approximation for the method in uniform norm.

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Antonio Cañete Universidad de Sevilla Spain antonioc@us.es

Antonio Cañete earned a degree in Mathematics from the University of Granada (Spain) in 2000. He finished his Ph.D. at the Geometry and Topology Department (University of Granada) in 2007, with a thesis entitled *Isoperimetric Problems in Surfaces*. His research interest is, mainly, the isoperimetric problem. He has been

teaching since 2005 in the University of León, University of Cádiz, and currently in University of Sevilla (Spain).

Title: The isoperimetric problem in surfaces of revolution

Abstract: Given a surface M, we may consider the isoperimetric problem consisting of finding the least perimeter set in M enclosing a certain value of the area. If such a set exists, then it is called isoperimetric region. This question has been completely solved only for certain surfaces, and recently treated in the setting of surfaces of revolution with some monotonic assumption on the Gauss curvature ([MHH], [R]).

In this talk we will study this problem in two large families of surfaces: symmetric tori of revolution with decreasing Gauss curvature from the largest-length parallel (including standard tori of revolution, [C]), and symmetric annuli of revolution with increasing Gauss curvature from the shortest-length parallel (including minimal catenoids and one-sheeted hyperboloids, [CR]).

It is well-known that any isoperimetric region is, in particular, a stable region (that is, a second order local minimum of perimeter under any variation preserving the area enclosed). In view of this fact, we will first follow the approach given in [R] in order to completely classify the stable regions in our surfaces. This classification will allow us to find the isoperimetric regions in these families of surfaces of revolution.

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Tom Carroll University College Cork Ireland t.carroll@ucc.ie http://euclid.ucc.ie/pages/staff/carroll/tcarroll.htm

Tom Carroll is a senior lecturer in mathematics at University College Cork in Ireland. He has worked with Prof. Rodrigo Bañuelos on isoperimetric-type problems for the exit time of Brownian motion and the principal frequency. His recent work includes results with Prof. Michiel van den Berg on 'Hardy inequality and  $L^p$  estimates for the torsion function' in Bulletin LMS and with Prof. Joaquim Ortega-Cerdà on 'The univalent Bloch-Landau constant, harmonic symmetry and conformal glueing' in Journal de Mathématiques Pures et Appliquées.

#### *Title: Interpolating between torsional rigidity and principal frequency*

Abstract: A one-parameter family of variational problems is introduced that interpolates between torsional rigidity and the first Dirichlet eigenvalue of the Laplacian.

The associated partial differential equation is derived, which is shown to have positive solutions in many cases. Isoperimetric results are obtained regarding extremal domains for fixed area and for convex domains of fixed inradius, as well as results regarding variation of the parameter. This is joint work with Jesse Ratzkin at Cork.





Laura Chasman Knox College USA lchasman@knox.edu faculty.knox.edu/lchasman

L. Mercredi Chasman currently resides in Illinois, USA, where she is a Visiting Assistant Professor at Knox College. She recently finished her Ph.D. at the University of Illinois at Urbana-Champaign, where she was a student of Richard Laugesen.

#### Title: An isoperimetric inequality for the free plate in all dimensions

Abstract: We establish an isoperimetric inequality for the fundamental tone (first nonzero eigenvalue) of the free plate of a given area, proving the ball is maximal in all dimensions and for all positive values of the tension parameter  $\tau$ . Given  $\tau > 0$ , the free plate eigenvalues  $\omega$  and eigenfunctions u are determined by the equation  $\Delta\Delta u - \tau\Delta u = \omega u$  together with certain natural boundary conditions. The boundary conditions are complicated but arise naturally from the plate Rayleigh quotient, which contains a Hessian squared term  $|D^2u|^2$ .

We adapt Weinbergers method from the corresponding free membrane problem, taking the fundamental modes of the unit ball as trial functions. These solutions are a linear combination of Bessel and modified Bessel functions.

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Anisa Chorwadwala is a Visiting Fellow at the School of Mathematics, Tata Institute of Fundamental Research in Mumbai, India. She received her B.Sc. in 1999, M.Sc. in 2001, and Ph.D. + M.Phil. in Mathematics in 2007 from University of Mumbai. Her PhD dissertation was entitled *Study of the Laplacian in a Class of Doubly Connected Domains on the Riemann Sphere*  $S^2$ . She is interested in Shape Optimization Problems.

#### Title: Placement of an obstacle as Shape Optimization Problem

Abstract: A typical Isoperimetric Problem is to enclose a given area A with a shortest possible curve. The classical Isoperimetric Theorem asserts that in the Euclidean plane the unique solution is a circle. There are many other results of a similar nature, referred to as Isoperimetric Inequalities of Mathematical Physics or sometimes as Shape Optimization Problems, where extrema are sought for various quantities of physical significance such as the energy functional or the eigenvalues of a differential equation. They are shown to be extremal for a circular or spherical domain. I will be talking about such Shape Optimization Problems.

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Hichem Chtioui Faculté des Sciences de Sfax Tunisia Hichem.Chtioui@fss.rnu.tn

Hichem Chtioui is Professor of Mathematics at the Faculté des Sciences de Sfax. He completed his PhD in 1998 at École Nationale d'Ingénieurs de Tunis (ENIT) under the supervision of Professor Abbas Bahri. His thesis was entitled: "Prescribing Scalar Curvature Problem".

#### Title: Prescribed Scalar Curvature Problem On Standard Spheres

Abstract: Our result is in the field of conformal geometry/geometry analysis. We provide a variety of classes of functions which can be realized as the scalar-curvature on the standard n dimensional sphere, n > 3 with respect to some conformal metric. Because of the presence of some critical nonlinearity, blow up phenomena occur, the existence results are highly nontrivial, since one has to overcome topological obstructions.

In this work, we extend the well-known existence results of Bahri-Coron and Chang-Yang. Our argument uses the theory of critical points at infinity introduced by A. Bahri.





Semra Demirel Universität Stuttgart semrademirel@web.de www.iadm.uni-stuttgart.de/LstAnaMPhy/Demirel

Semra Demirel is a currently completing a PhD degree working under the supervision of Professor Timo Weidl.

Title: Inequalities for eigenvalues of quantum graphs

Abstract: We study the spectra of quantum graphs with the method of trace identities (sum rules), which are used to derive inequalities of Lieb-Thirring, Payne-Pólya-Weinberger, and Yang types, among others. We show that the sharp constants of these inequalities and even their forms depend on the topology of the graph. We also provide some counterexamples where the classical form of the inequalities is false.





Brahim Dridi Faculté des Sciences de Gabès Tunisia dridi\_br@yahoo.fr

Brahim Dridi is Maître Assistant of Mathematics at the Faculty of Sciences of Gabès. He defends his Ph.D thesis in PDE's and Geometry in June 2010. His Thesis, entitled "Quelques problèmes liés la géométrie Kählérienne was supervised by Professor Adnène Ben Abdesselem and Professeur Saoussen Kallel-Jallouli.

Title: Dirichlet problem for degenerate elliptic Monge-Ampère equation relative to a Hermitian metric.

Abstract: We consider the Dirichlet problem for a complex Monge-Ampère equation:

$$\det \nabla^{\lambda}{}_{\mu} \phi = f \text{ in } M$$
$$\phi = u \text{ on } \partial M, \qquad (1)$$

where M is a  $C^{\infty}$  strongly pseudoconvex hermitian compact manifold with boundary, f is a smooth nonnegative function satisfying certain hypotheses and u a smooth function. We prove the existence of a  $C^{1,\alpha}$ ,  $\alpha \in (0, 1)$ , solution for the problem

[Joint work with Professor Saoussen Kallel-Jallouli]

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Bodo Dittmar Martin-Luther-University Halle-Wittenberg, Germany bodo.dittmar@mathematik.uni-halle.de http://www2.mathematik.uni-halle.de

- PhD in Geometric Function Theory, 1979 "Extremal problems of the Schiffer-Hawley type for conformal and quasiconformal mappings" under the direction of Prof. R. Kühnau
- Postdoctoral Thesis, 1988 "Stekloff eigenvalues and conformal mappings"
- Full Professor at the Martin-Luther-University Halle-Wittenberg 1992

#### Title: Sums of reciprocal eigenvalues

Abstract: The aim of the talk is to present a general approach to derive formulas and isoperimetric inequalities for sums of reciprocal eigenvalues (Math. Nachr. 237 (2002), 268(2004)). This method works generally for fixed and elastically supported membrane eigenvalues and also for different kinds of Stekloff eigenvalue problems and is restricted to domains in the plane. On the one hand we are able to calculate exactly the sum over all reciprocal eigenvalues if the conformal mapping between the unit disc and the domain under consideration is known on the other hand isoperimetric inequalities follow. Among others the following result has been obtained: It was proved by Polya and Schiffer 1954 that the harmonic means of the reciprocal eigenvalues of the fixed membrane is minimal for the disc for simply connected plane domains with maximal conformal radius 1 and now it follows from the mentioned approach that the same is true for the positive free membrane eigenvalues (Journal Anal. 95(2007), ZAMP 60(2009)). Another result following with this method is a sharper version of the Pólya-Schiffer inequality.



Rebecca Dorff Brigham Young University USA beccadorff@gmail.com

Rebecca is one of three students, (along with Donald Sampson and James Dilts) that have been working with Dr. Gary Lawlor of Brigham Young University on Multiple Bubble problems. These students have pioneered new applications of metacalibration theory to isoperimetric problems. Rebecca enjoys many hobbies, including fencing and cross-stitching.

#### Title: An Introduction to Metacalibration and its Application to Multiple Bubble Problems

Abstract: Metacalibration is an exciting new method in geometric optimization authored by Gary Lawlor of Brigham Young University. It is a generalization of calibrated geometries. We present metacalibration theory and discuss its application to multiple bubble problems. These problems ask which gure minimizes boundary among all gures that separately enclose a given set of volumes.

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Salem Eljazi Faculté des Sciences de Tunis Tunisia eljazisalem@gmail.com

Salem Eljazi is completing a PhD degree under the supervision of Professor Najoua Gamara. He is currently Assistant at the Institut Préparatoire aux Etudes d'Ingénieur de Gafsa. His research interests revolve around Riemannian and CR geometries.

Title: The Webster scalar curvature revisited: Case of the three dimensional CR sphere

Abstract: We consider the problem of prescribing the Webster scalar curvature on the three CR sphere of  $\mathbb{C}^2$ . This problem is equivalent to solving the following semi-linear equation:

$$\begin{cases} L_{\theta_1} u = K \ u^{1+\frac{2}{n}} & \text{on } \mathbb{S}^{2n+1} \\ u > 0 \end{cases}$$

$$\tag{2}$$

Here L is the conformal laplacian of  $\mathbb{S}^{2n+1}$ ,  $L_{\theta_1} = (2 + \frac{2}{n})\Delta_{\theta_1} + R_{\theta_1}$ , where  $\Delta_{\theta_1} = \Delta_{\mathbb{S}^{2n+1}}$ and  $R_{\theta_1} = \frac{n(n+1)}{2}$  are respectively the sublaplacian operator and the Webster scalar curvature of  $(\mathbb{S}^{2n+1}, \theta_1)$ .

We use techniques related to the theory of critical points at infinity, and obtain existence results for curvature satisfying an assumption of Bahri-Coron type.

Luigi Fontana Università di Milano-Bicocca, Italy luigi.fontana@unimib.it

Born in Lodi (Italy) in 1960, he received his Ph.D in Mathematics from Washington University in St. Louis in 1991. He got a position at the Università di Milano from 1992 to 1999 and since then, he works at the Università di Milano-Bicocca. His main research interests are in harmonic analysis, exponential integrability and several complex variables.

Title: Moser-Trudinger inequalities on the CR-sphere

Abstract: I will present results concerning sharp exponential integrability for powers of the sublaplacian and for a rather general family of operators acting on CR-pluriharmonic functions. Such results were the key to the proof of the sharp Beckner-Onofri inequality on the CR-sphere. This is part of joint works with Carlo Morpurgo, and C. Morpurgo - T. Branson.



Leander Geisinger Universität Stuttgart Germany Leander.Geisinger mathematik.uni-stuttgart.de www.iadm.uni-stuttgart.de/LstAnaMPhy/Geisinger

Leander Geisinger is a currently completing a PhD degree working under the supervision of Professor Timo Weidl.

Title: Universal Bounds for Traces of the Dirichlet Laplace Operator

Abstract: We derive upper bounds for the trace of the heat kernel Z(t) of the Dirichlet Laplace operator in an open set  $\Omega \subset \mathbb{R}^d$ ,  $d \geq 2$ . In domains of finite volume the result improves an inequality of Kac. Using the same methods we give bounds on Z(t) in domains of infinite volume. To prove the results we employ refined Berezin-Li-Yau inequalities for eigenvalue means.

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Dinara Giniyatova Kazan State University Russia normaliti@gmail.com

Dinara Kh. Giniyatova was born in Kazan in 1985. She is a graduate student at the "Chair of Theory of Functions and Approximations", Kazan State University (Russia) working with Professor Farit G. Avkhadiev. The sphere of her interests include isoperimetric inequalities in mathematical physics (particularly, inequalities relating with the torsional rigidity), Schwarz-Pick type inequalities, coefficient estimates and estimates for derivatives of conformal radius. She is versatile in both English and French, participates in sports, and likes to read Science Fiction books.

#### Title: Torsional rigidity and Euclidean moment of inertia

Abstract: We consider the torsional rigidity problem for the elastic uniform beam with a simply connected section. It is proved (Avkhadiev F.G., 1998), that torsional rigidity is equivalent to the geometric functional as the conformal moment and the moment of the domain about its boundary. Let  $P(\Omega)$  be the torsional rigidity of  $\Omega$  and  $I(\Omega)$  be the moment of  $\Omega$  about its boundary. Then, as is known  $P(\Omega) < 4I(\Omega)$  for convex domains. But this estimate is not valid for simply connected domains (Kovalev L.V., 2002). Let  $C_0$  is the optimal constant in the class of simply connected domains, then  $C_0 \leq 64$ . We show, that  $C_0 > 4.08$ . Obviously, this constant may be improved, if we will find the domain for that  $P(\Omega)/I(\Omega)$  will rank over the referred estimate. But the finding of these domains is difficult because we can't write the distance function

referred estimate. But the finding of these domains is difficult because we can't write the distance function for arbitrary simply connected domains in explicit form. And with it one can consider analogous problem for the conformal moment of  $\Omega$ .

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Alexandre Girouard Institut de Mathématiques de Neuchâtel, Suisse alex.girouard@gmail.com members.unine.ch/alexandre.girouard

Alexandre Girouard completed his PhD at the University of Montreal, Canada in 2008 working under the supervision of Professor Iosif Polterovich and Professor Marlène Frigon with a thesis entitled *Degeneration and extremal problems for eigenvalues of the Laplace operator on surfaces.* He spent the 2008-2009 year as a Postodoctoral Research Associate at the Cardiff School of Mathematics. He is currently a Maître Assistant at the Institut de Mathématiques de Neuchâtel.

#### Title: Shape optimization for lower eigenvalues of the Neumann and Steklov problems

Abstract: The Pólya conjecture (1954) states that the k-th Neumann eigenvalue of a planar domain is bounded above by  $4k\pi$ . In this talk I will present a sharp isoperimetric inequality for the second nonzero eigenvalue. This implies the Pólya conjecture for k = 2. I will also discuss similar results for the spectrum of the Dirichlet-to-Neumann map and for the spectrum of the Laplace-Beltrami operator on spheres. Surprisingly, this extension to spheres is possible only for odd dimensions. This is joint work with Iosif Polterovich and Nikolai Nadirashvili.





Nabil Gmati LAMSIN-ENIT, Tunis, Tunisia nabil.gmati@ipein.rnu.tn www.lamsin.rnu.tn

Nabil Gmati was born in 1962, in Tunis. Professor at ENIT (Ecole Nationale d'Ingénieurs de Tunis) and Head of LAMSIN (Laboratoire de Modélisation Mathématique et Numérique pour les Sciences de l'Ingénieur), Tunis, Tunisia. His research activity has been mainly related to numerical methods for acoustic wave propagation in unbounded domains.

#### Title: Total Overlapping Schwarz Preconditioners

Abstract: Carrying relevant simulations of coupled models generally requires computational resources allowing to zoom on the regions of small scales. A significant part of the applications include fluid-particle flows such as blood flow, structural mechanics with components having different (geometrical, physical) scales factors or fracture mechanics, cracks propagation and material fatigue. Basically, Domain Decomposition methods are an affordable computational multi-scale methodology to handle those problems in a satisfactory way. Their use may be either as a tool to match multi-resolution approximations built on grids liable to embrace the local characteristics of the problem or as a powerful process to design performing sub-structuring preconditioners to solve the discrete system. Letting aside the most classical and popular domain decomposition approaches, recent attempts concentrate on the opportunity of coupling global computations with a zoom in where small structures need to be simulated.

We proposed in a previous work, a variant of the Totally Overlapped Schwarz Method (TOSM) to approximate iteratively the exact absorbing boundary condition, for exterior problems. The same method may be an aiding tool for numerical zooming on regions of a particular interest. TOSM enjoys, then, the ability in computing small structures one wants to capture and the efficiency in obtaining, at a reasonable cost, the behavior of the solution far from the obstacles for infinite domains. The main aim of this work is to use this modified Schwarz procedure as a preconditioner to Krylov subspaces methods so to accelerate the calculations. A study provide the convergence rate of the Schwarz preconditioned GMRES method in two and three dimensions.

Joint work with Faker Ben Belgacem (Laboratoire de Mathématiques Appliquées de Compiègne, Université de Technologie de Compiègne) and Faten Jelassi (LAMSIN and Faculté des Sciences de Bizerte).



Samah Horrigue Institut supérieur des systèmes industriels, Gabès, Tunisia horrigues@yahoo.fr

Samah Horrigue completed her PhD at the Faculté des Sciences de Tunis in 2009 under the supervision of Professor Habib Ouerdiane. Her PhD dissertation was entitled "Hankel Operators and Quantum Gross Laplacian". Her research interests include the evolution equation associated with the quantum Gross Laplacian and its powers, Generalized Hankel operators, and stochastic processes.

#### Title: Rotation-invariant of Quantum Laplacians

Abstract: In this paper, we prove that the Quantum Gross-Laplacian and the Quantum Beltrami-Laplacian denoted respectively  $\Delta_{QG}$  and  $\Delta_{QB}$  are a rotation-invariant operators. For this purpose, we use the Schwartz-Grothendieck kernel theorem and the characterization theorem of rotation-invariant distributions and operators. Then, we give a characterization of all Quantum operators by means of rotation-invariance.





Ryan Hynd received his Ph.D. in May 2010; his dissertation addresses various partial differential equation, free boundary problems arising in singular control theory and mathematical finance. Ryan will be a Courant Instructor at NYU in the Fall of 2010 and his research interests include partial differential equations, differential geometry, mathematical finance and problems involving optimization.

Title: Symmetric constant mean curvature surfaces in  $\mathbb{S}^3$ 

Abstract: We introduce a notion of symmetry in  $\mathbb{S}^3$  that is a natural generalization of rotational symmetry in  $\mathbb{R}^3$ . We classify the CMC surfaces having this symmetry and find that they bear a strong resemblance to the Delaunay surfaces in  $\mathbb{R}^3$ . In particular, we obtain explicit parametrizations of families of embedded CMC tori in  $\mathbb{S}^3$ .



Mette Iversen University of Bristol United Kingdom mette.iversen@bristol.ac.uk

Mette Iversen is a native of Denmark. She moved to England in 2005 to pursue her studies. She graduated from the University of Bristol in 2008 and spent the following year completing the CASM in Cambridge. In August 2009, she returned to Bristol to start a PhD. Her supervisor is Professor Michiel van den Berg, with whom she is working on problems centered around shape optimisation and spectral theory.

#### Title: Minimisers of Dirichlet Eigenvalues with Geometric Constraints

Abstract: This talk considers the problem in shape optimisation of minimising the eigenvalues of the Dirichlet Laplacian in  $\mathbb{R}^m$  subject to geometric constraints on the minimising set. In particular, it will look at the number of components of the minimising set subject to a constraint on the Hausdorff measure of the boundary.

This is work with Professor Michiel van den Berg from the University of Bristol.



Asma Jbilou Université de Nice-Sophia Antipolis, France Asma.Jbilou@unice.fr math.unice.fr/~jbilou

Asma Jbilou is currently a Lecturer and Researcher at the University of Nice Sophia-Antipolis, France.

Jbilou was born in Casablanca, Morocco in May 1982. She left Morocco in 2000 to pursue university studies in France: Classes préparatoires at the Lycée Louis-le-Grand (Paris), Licence and Master in Pure Mathematics at Pierre et Marie Curie-Paris VI University. Jbilou completed her Master thesis in 2006 under the direction of Professor Paul Gauduchon (Directeur de Recherches, CNRS) on *The positive mass conjecture by the spinorial method of Witten*.

She moved to Nice in 2006, where she received her *Doctorat* (Ph.D.) in February 2010, from University of Nice Sophia-Antipolis under the supervision of Professor Philippe Delanoë (Directeur de Recherches, CNRS).

Her Ph.D. dissertation deals with Hessian equations on some compact Kähler manifolds the results of which were recently announced in an article of the Comptes Rendus de l'Académie des Sciences de Paris.

Asma Jbilou is interested in compact Kähler manifolds, equations of Monge-Ampère type (real and complex ones), Hessian equations (real and complex ones) and uniformly elliptic P.D.Es of second order, in general.

#### Title: Complex Hessian equations on some compact Kähler manifolds

Abstract: On a compact connected 2m-dimensional Kähler manifold with Khler form  $\omega$ , given a volume form  $\Omega \in [\omega]^n$  and an integer 1 < k < m, we want to solve uniquely in  $[\omega]$  the equation  $\tilde{\omega}^k \wedge \omega^{m-k} = \Omega$ , relying on the notion of k-positivity for  $\tilde{\omega} \in [\omega]$  (the extreme cases are solved: k = m by Yau, k = 1 trivially). We solve by the continuity method the corresponding complex elliptic k-th Hessian equation under the assumption that the holomorphic bisectional curvature of the manifold is non-negative, required here only to derive an a priori eigenvalues pinching.



Moez Khenissi ESST Hammam Sousse Tunisia moez.khenissi@gmail.com

Moez Khenissi was born in Zarzis in 1972. He obtained his PhD from the Faculty of Sciences of Tunis in 2001 with a thesis entitled *Equations des ondes amorties dans un domaine extrieur et calcul du meilleur taux*, and his HDR in 2009 on *Problèmes de stabilisation pour des E.D.P.* from the Faculty of Sciences of Monastir. He is maître de conferences at the École Supérieure des Sciences et Technologie de Hammam Sousse.

#### Title: On the Resolvent Estimates of some Evolution Equation and Applications

Abstract: The purpose of this talk is to study the distribution of the resonances for some models of damped equations in exterior domains under some natural assumption on the support of the damping term. As consequence we obtain a local energy estimates of their solutions.



Born in 1984 in Cuneo, Italy, Andrea Mondino received hi B.S. in 2007 at the University of Turnin (supervisor Prof. Fino), M.S. at SISSA (supervisor Prof. Malchiodi). Since October 2008, he has been pursuing a PhD degree at SISSA working under the supervision of Professor Andrea Malchiodi.

Title: The Willmore Functional: a perturbative approach

Abstract: The Willmore functional is a natural quantity associated to a closed surface immersed in  $\mathbb{R}^3$  or more generally in a Riemannian manifold: it is defined as the integral of the squared mean curvature. A classical problem is to find surfaces that minimize (or more generally extremize) the functional, called Willmore surfaces. These surfaces satisfy a forth order nonlinear elliptic PDE.

Using a perturbative method which relies on a finite dimensional reduction procedure, we show the existence of Willmore surfaces immersed in  $\mathbb{R}^3$  endowed with a metric closed and asymptotic to the euclidean one.





Tatiana Odzijewicz is a student in the Doctoral Program in Mathematics and Applications at the University of Aveiro in Portugal. Most of her research interests center around fractional calculus of variations and its generalizations.

#### Title: Isoperimetric problems for generalized fractional operators

Abstract: We study isoperimetric problems of the calculus of variations with more general fractional operators. Necessary optimality conditions of Euler-Lagrange type for three different types of generalized fractional operators are considered. New results are obtained even in the particular case when the generalized operators are reduced to be the standard fractional derivatives and fractional integrals in the sense of Riemann-Liouville or Caputo.





Mihai Pascu Transilvania University of Brasov Romania mihai.pascu@unitbv.ro cs.unitbv.ro/~pascu

Mihai Pascu received his B.S. and M.S. from Transilvania University of Brasov (1990-1996) in Romania, M. S. and Ph. D. from University of Connecticut at Storrs (1996-2001). He held a postdoctoral position at Purdue University (2001-2004) and has been, since 2004, Associate Professor at Transilvania University of Brasov. His research interests are: Brownian motion (finite and infinite dimensional), relating monotonicity properties (e.g. Hot spots conjecture, Laugesen-Morpurgo conjecture, Chavel's conjecture, maximum principles), monotonicity properties of densities of reflecting Brownian motion in smooth domains.

#### Title: Mirror couplings of reflecting Brownian motions and applications

Abstract: In a series of papers, Burdzy et. al. introduced the mirror coupling of reflecting Brownian motions in a smooth bounded domain  $D \subset \mathbb{R}^d$ , and used it to prove certain properties of eigenvalues and eigenfunctions of the Neumann Laplacian on D.

In this talk we show that the construction of the mirror coupling can be extended to the case when the two Brownian motions live in different domains  $D_1, D_2 \subset \mathbb{R}^d$ . As an application of the construction, we derive a unifying proof of the two main results concerning the validity of Chavel's conjecture on the domain monotonicity of the Neumann heat kernel, due to I. Chavel (J. London Math. Soc., 1986), respectively W. S. Kendall (J. Funct. Anal., 1989).





Gerard Philippin Université Laval, Québec Canada gphilip@mat.ulaval.ca

- Ph.D. at the ETH-Zűrich, (Switzerland)
- Postdoctoral fellow at Waterloo University (Canada) and at Cornell University (USA)
- Professor at the Department of mathematics at Laval University (Canada)

Title: Some isoperimetric inequalities with applications to the Stekloff problem

Abstract: Isoperimetric inequalities for the product of some moments of inertia will be presented. As an application, an isoperimetric inequality for the product of the N first eigenvalues of the Stekloff problem in  $\mathbb{R}^N$  will be derived.

Reference: A. Henrot, G. A. Philippin, A. Safoui, Some isoperimetric Inequalities with application to the Stekloff problem, 15, (2008), pp. 581-592.



Jesse Ratzkin University College Cork Ireland j.ratzkin@ucc.ie

Ph.D. 2001 from the University of Washington, under the direction of Daniel Pollack. Post-doctoral positions at U. Utah, U. Connecticut, U. Georgia, and University College Cork.

Recent relevant papers:

A Payne-Weinberger eigenvalue estimate for wedge domains on spheres (with A. Treibergs), Proc. Amer. Math. Soc. 137 (2009), 2299-2309.

Eigenvalues of Euclidean wedge domains in higher dimensions, arXiv:AP/0905.0199.

Interpolating between torsional rigidity and principal frequency (with T. Carroll), arXiv:AP/1003.0128. Isoperimetric inequalities and variations on Schwarz's Lemma (with M. van den Berg and T. Carroll), in preparation.

Title: Isoperimetric inequalities and variations on Schwarz's Lemma

Abstract: The classical Schwarz lemma states that if f is an analytic function from the unit disk to itself then  $|f(z)| \leq |z|$  with equality if and only if f is a rotation map. I will discuss a version of Schwarz's lemma for the first Dirichlet eigenvalue of the Laplacian, and some ingredients in its proof. The key ingredient is a new isoperimetric inequality for the first eigenfunction: if we give a simply connected domain D a new conformal metric ds = |du||dz|, where u is the first eigenfunction, then  $L^2 \geq 4\pi A$ , with equality if and only if D is a disk.



Dinar arabe de Carthage



Ali Saada LAMSIN-ENIT Tunisia Ali.Saada@ipein.rnu.tn www.lamsin.rnu.tn

Ali Saada received his PhD in Mechanics at INSA-Lyon in 1992. His thesis was on a "Mathematical model for epicyclical trains". He was Assistant Professor of Mathematics at the Nabeul Engineering Preparatory School (Tunisia), 1993-2005, and more recently, Assistant Professor of Mathematics at the Higher School of Communication of Tunis (Sup'Com), 2005-present. He received his Habilitation in 2009. Responsible for the CMCU project "Numerical methods in hydrogeology", 2004-2008, and Responsible for the Associate Team MODESS ("Mathematical models of flows in fractured porous media"), 2005-2009. Saada developed the "Numerical analysis and optimization" course at Sup'Com which he has been teaching since 2005.

He is interested in developing models to predict subflows in 3D complex geometry using domain decomposition methods to compute the discrete solution. These models can be used for environmental problems and by petroleum companies.

#### Title: A Single Flow Phase Model in a Medium with Known Fractures

Abstract: In this talk I will present single phase flow model in a medium with known fractures. In the rock matrix the flow is governed by Darcy's law and also in some fractures. In other fractures, when the flow rate is large, the model use the non linear Forchheimer's law. I will describe a model in which the fractures are treated as interfaces. I will also consider the case of intersecting fractures and the case of nonconforming meshes.



Rustem Salakhudinov Kazan State University Russia Rustem.Salahudinov@ksu.ru

Rustem Salakhudinov received his PhD in Mathematics and Physics from Kazan State University, Russia in 1998 with a dissertation entitled: "Isoperimetric inequalities for moments of inertia of plane domains". He is Docent by specialization in Mathematical Analysis. He received the VAK Certificate from the Supreme Certifying Committee in Moscow, Russia (2006). Salakhudinov carrier includes the position of Researcher (1995-2001) and Senior Researcher (2001-present) at N.G. Chebotarev Institute of Mathematics and Mechanics at Kazan State University Assistant Professor (2002-2008) at the Department of Mathematical Analysis, and Assistant Professor at the Department of General Mathematics, Faculty of Calculation Mathematics and Cybernetics of KSU (2004-2007), Docent at the Department of Economy, KSU (2007-2008), and Docent, Department of Mathematical Analysis, Mechanics and Mathematics, KSU, Russia (2008-present).

#### Title: Integral properties of the warping function of plane domains

Abstract: Let G be a multiple connected plane domain. Let us introduce the following physical functional

$$\Phi(p) := \frac{1}{u(G)^{p+1}} \left( T_p(G) - \frac{2\pi u(G)^{p+1}}{p+1} \right),$$

where u(x, G) is the classical warping function of  $G, p \ge 0$ , and

$$T_p(G) = \int_G u(x,G)^p \, dA, \quad u(G) := \sup_{x \in G} u(x,G).$$

L.E. Payne proved that, the excretion in the brackets is not negative, and it turns identically zero iff G is a concentric ring.

**Theorem.** Let G is a multiply connected domain, and  $T_{p_0}(G) < +\infty$  for some  $p_0 \in [0,\infty)$ . Then

1) If G is not a concentric ring, then  $\Phi(p)$  is a strictly decreasing function of p.

2) If G is a concentric ring, then  $\Phi(p) \equiv 0$ , for  $p \in [0, +\infty)$ .

In particular, we get the inequality  $\Phi(1) \leq \Phi(0)$ , which is equivalent to the classical Payne inequality for torsional rigidity of G. We also discus several similar problems for integrals of the warping function.

This work was partially supported by the Russian Foundation for Bassic Research, project no. 08-01-00381-a.



Donald Sampson Brigham Young University USA sampson.dcs@gmail.com

Donald is one of three students, (along with Rebecca Dorff and James Dilts) that have been working with Dr. Gary Lawlor of Brigham Young University on Multiple Bubble problems. These students have pioneered new applications of metacalibration theory to isoperimetric problems. Donald was married on year ago and has a new daughter!

Title: An Introduction to Metacalibration and its Application to Multiple Bubble Problems

Abstract: Metacalibration is an exciting new method in geometric optimization authored by Gary Lawlor of Brigham Young University. It is a generalization of calibrated geometries. We present metacalibration theory and discuss its application to multiple bubble problems. These problems ask which gure minimizes boundary among all gures that separately enclose a given set of volumes.



Ilnar Shafigullin Kazan State University Russia Shafigullin.IK@gmail.com

Shafigullin Ilnar was born in 1987 in Russia. He successfully graduated from Gymnasium in 2004, then from the Faculty of the Mechanics and Mathematics (Department of Theory of Functions and Approximations) in 2009, and Faculty of Economics (Department of Management) in 2010, at Kazan State University (Russia). For three years, he was Captan of the university's chess team. He likes to take part in any kinds of intellectual games and competitions. In his spare time, he likes to play classical guitar. He is a first-year postgraduate student working on Hardy-type inequalities under the supervision of Professor F.G. Avkhadiev.

Title: Isoperimetric characteristics of domains and variational inequalities of mathematical physics

Abstract: We consider a family of the Hardy-type inequalities, which includes three integral terms in special planar domains. If isoperimetric profile of the domain belongs to a special class of functions, than the inequalities are valid and there is two-sided estimate of their constants. We also study the constants which characterize isoperimetric profile. In particular, we establish correlation of these constants at the spaces X and  $X \times \mathbb{R}^n$ .

This research is supported by Federal Target Program "Scientific and Scientific-Pedagogical Personal of Innovation Russia" (contract No 02.740.11.0193).





Bartlomiej Siudeja University of Illinois USA siudeja@illinois.edu www.math.uiuc.edu/~siudeja

Bartlomiej Siudeja received his Ph.D. in 2008 from Purdue University under supervision of Professor Rodrigo Bañuelos. At present (2008-2011) he is a J. Doob Research Assistant Professor at the University of Illinois at Urbana-Champaign. He works under guidance of Professor Richard Laugesen. His research interests consist of potential analysis of stable processes and spectral analysis of local and non-local operators related to stochastic processes.

Title: The method of rotations and tight frames: sharp upper bounds on Laplace eigenvalues

Abstract: Extremal domains often possess rotational symmetry, for example, equilateral triangles, squares and disks, or in higher dimensions the regular simplexes, cubes and balls. We show how to exploit this symmetry to prove sharp upper bounds on sums of eigenvalues of the Laplacian.

The method applies to competing domains that are linear transformations of the extremal. Thus our results cover all triangles (simplexes) and parallelograms (parallelepipeds). To extract the effect of the linear transformation on the Rayleigh quotient, we average over the rotation group of the extremal domain and employ the tight frame property of its orbits.

The competing domains must be normalized, of course. We employ a natural normalization involving moment of inertia and area.

Notable features of the method are that it applies for all major boundary conditions (Dirichlet, Robin or Neumann), for eigenvalue sums of arbitrary length, and that it requires no explicit knowledge of the eigenvalues or eigenfunctions of the extremal domain.

The talk will illustrate this Method of Rotations and Tight Frames with several examples, such as that the scale-invariant eigenvalue sum

$$(\lambda_1 + \dots + \lambda_n) \frac{A^3}{I}$$

is maximal among triangles for the equilateral triangle, for each  $n \ge 1$ .

[Joint work of R. S. Laugesen and B. A. Siudeja.]



Pedro Solórzano Stony Brook University USA pedro@math.sunysb.edu www.math.sunysb.edu/~pedro

Pedro Solórzano is currently finishing his PhD in Mathematics at the State University of New York at Stony Brook (SUNYSB). He worked under the supervision of late Detlef Gromoll. After his passing he continued his work under the supervision of H. Blaine Lawson Jr. (SUNYSB) and Christina Sormani (CUNY-GC). He go his Bachelor's degree from UNAM in Mexico City. His current interests are in Riemannian geometry from a synthetic approach. He is working on topics relating Gromov-Hausdorff convergence and holonomy.

Title: An isoholonomic approach to Riemannian geometry

Abstract: On a 2-dimensional connected Riemannian manifold, the isoperimetric problem can be reformulated as an isoholonomic problem by considering a connection whose curvature form is a multiple of the volume form.

In general, given a complete Riemannian manifold M, a Sasaki-type metric turns the total space of a vector bundle over M into a complete Riemannian manifold. With this metric, the individual fibers are totally geodesic flat submanifolds. As such, their induced length metric is Euclidean. Furthermore, the global geometry of M becomes apparent when considering the restricted (i.e. subspace) metric of the fibers from that of the total space. The linear structure on individual fibers, the holonomy action, and the isoholonomic problem all interact.

I will discuss these constructions and mention some applications.



Alina Tukhvatullina Kazan State University Russia Alina.Tukhvatullina@ksu.ru

Alina Tukhvatullina was born in Russia in 1987. She successfully graduated from the Faculty of Mechanics and Mathematics of Kazan State University, Russia in 2009. She is a first-year graduate student working on Hardy-type inequalities under the supervision of Professor F.G. Avkhadiev.

#### Title: Hardy-type inequalities for a special family of non-convex domains

Abstract: We prove Hardy type inequalities involving the distance to the boundary and the volume of domain for a large family of domains which are not convex. Namely, we obtain multidimensional Hardy type inequalities for non-convex regular domains. These inequalities are similar to inequalities which were obtained by M. Hoffmann-Ostenhof, T. Hoffmann-Ostenhof and A. Laptev for convex domains. Moreover, we obtain the sufficient condition for regularity of domains. This condition is a generalization of E. B. Davies' result (1995). We also give a number of examples of regular domains with sharp constants of regularity and present Hardy type inequalities for these domains.

This research is supported by Federal Target Program "Scientific and Scientific-Pedagogical Personal of Innovation Russia" (contract N02.740.11.0193)





Na Wei Université de Toulouse 1 France nawei8382@gmail.com

Na Wei is a Ph. D student in Northwestern Polytechnical University of China. She is currently affiliated with the Centre de Recherche Mathematique de l'Université Toulouse 1, Toulouse, France, as a joint training Ph.D student. Her research topic in France revolves around "The role of eigenvalues for some elliptic problems".

Title: Dirichlet Eigenvalue Ratios for the p-sub-Laplacian in the Carnot Group

Abstract: We prove some new Hardy type inequalities on the bounded domain with smooth boundary in the Carnot group. Several estimates of the first and second Dirichlet eigenvalues for the p-sub-Laplacian are established.

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Khaled Zennir Université des Sciences et de la Technologie, Oran, Algeria khaledzennir2@yahoo.com

Khaled Zennir was born in 1982 in Skikda, Algeria. In 2006, he completed his Diploma of Graduate Studies (DES) in mathematics at the Université "20 Août 55" de Skikda specializing in Numerical Analysis. He defended his Masters' degree in February 2009 at the Badji Mokhtar University of Annaba with a concentration in Dynamical Systems and Functional Analysis. His research interests include Non Linear Viscoelastic Hyperbolic Equations.

#### Title: Local existence and exponential growth for a nonlinear viscoelastic hyperbolic equation

Abstract: In this work, we consider a viscoelastic wave equation, with strong damping, nonlinear damping and source terms . subjected to the initial and boundary conditions in bounded domain in  $\mathbb{R}^N$ , with smooth boundary ; with the presence of positives constants, and nonnegative, non-increasing function. This type of problems are not only important from the theoretical point of view, but also arise in many physical applications and describe a great deal of models in applied science. One of the most important eld of such problems arise in the models of nonlinear viscoelasticity. Many authors studied these types of problems, and several results appeared in the literature. The goal of this work is the study of the local existence and exponential growth of solutions of the problem.



Mourad Zeraï ESPRIT and LAMSIN-ENIT Tunisia mourad.zerai@gmail.com

Mourad Zeraï teaches at the École Supérieure Privée d'Ingénierie et de Technologies (ESPRIT). He is a research associate at the Laboratoire de Modélisation Mathématique et Numérique pour les Sciences de l'Ingénieur (LAMSIN) of the École Nationale d'Ingénieurs de Tunis (ENIT).

#### Title: Riemannian Image Processing

Abstract: We present a novel approach for the derivation of partial differential equations used in analysing and processing digital images. The constrained nature of data describing an image leads us to consider the Riemannian geometry as a natural setting for image processing. We endow the manifold-valued image by an appropriate metric and we derive the differential geometric attributes, such as the covariant derivative and the Christoffel symbols. Then we use these materials to extend scalar-valued methods to the manifoldvalued image framework. Experiments on synthetic and real images show that the proposed Riemannian image processing approach is effective and highly robust.



### PARTICIPANTS

NAME Abdelhedi Wael Abramov Denis Ait Haddou Hassan Amri Amine Aouadi Saloua Aribi Amine Ashbaugh Mark Atallah-Baraket Amel Avkhadiev Farit Azzouz Abdelhalim Bahri Sidi Mohammed Bandle Catherine Baniata Mashhour Bañuelos Rodrigo Baraket Sami Belhaj Rhouma Nedra Bel Kenani Toukabri Lamia Ben Ahmed Ali Ben Amor Ali Benguria Rafael Benguria Soledad Ben H. Rhouma Mohamed Benmoussa Mohammed Tayeb Ben Naceur Raouf Bennaoum Malika Ben Omrane Inès Ben Othmane Sonia Bianchini Chiara Bouchriha Habib Bouhadjera Hakima Boulaaras Salah Burchard Almut Cádiz Fabián Cañete Antonio Carroll Tom Chasman Laura Chemikh Smail Chemmam Rym Chorwadwala Anisa Chtioui Hichem Cianchi Andrea Colbois Bruno Dammak Makkia De Castro Yohann Demirel Semra Dilts James Dittmar Bodo Dorff Michael Dorff Rebecca Douchich Ikmar Dridi Brahim Druet Olivier El Jazi Salem El Soufi Ahmad Fantar M. Hassine Fardoun Ali Ferone Vincenzo Fleckinger-Pellé Jacqueline Fontana Luigi Frank Rupert Friedlander Lennie Gamara Najoua Geisinger Leander Ghomi Mohammad Giniyatova Dinara Girouard Alexandre Gmati Nabil Guemri Habiba Hajjej Mohamed Hamza Mohamed Ali Harrabi Abdelaziz

AFFILIATION Faculté des Sciences de Sfax Kazan State University I. R. I. de Toulouse Faculté des Sciences de Tunis Faculté des Sciences de Tunis Faculté des Sciences de Tunis University of Missouri Faculté des Sciences de Tunis Kazan State University Université de Saida Faculté des Sciences de Mostaganem University of Basel Mutah University Purdue Universitv King Saud University Faculté des Sciences de Tunis Faculté des Sciences de Tunis Faculté des Sciences de Tunis Faculté des Sciences de Gafsa P. Universidad Católica de Chile U. Wisconsin-Madison Sultan Qaboos University Université de Ouargla Faculté des Sciences de Tunis University of Oran Faculté des Sciences de Faculté des Sciences de Tunis U. Henri Poincaré, Nancy 1 Faculté des Sciences de Tunis Badji-Mokhtar U. of Annaba I.H.F.R. d'Oran University of Toronto P. Universidad Católica de Chile Universidad de Sevilla University College Cork Knox College Université H. Boumediene Faculté des Sciences de Tunis Tata Institute of Fundamental Research Faculté des Sciences de Sfax Università di Firenze Université de Neuchâtel I.P.E.I. de Bizerte Institut de Mathématiques de Toulouse Universität Stuttgart Brigham Young University Martin-Luther-U. Halle-Wittenberg Brigham Young University Brigham Young University Faculté des Sciences de Bizerte Faculté des Sciences de Tunis école Normale Supérieure de Lyon I.P.E.I. de Gafsa Université de Tours Chaire du Président Ben Ali Université Brest Università degli Studi di Napoli "Federico II" Université de Toulouse Università di Milano-Bicocca Princeton University University of Arizona Faculté des Sciences de Tunis Universität Stuttgart Georgia Institute of Technology Kazan State University Université de Neuchâtel LAMSIN-ENIT Faculté des Sciences de Tunis Université Blaise Pascal Faculté des Sciences de Tunis I.M.A.I. de Kairouan

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wael\_hed@yahoo.fr quaerix@gmail.com haithaddou@yahoo.com amineamri.fst@gmail.com saloua.mani@fst.rnu.tn aminemth@yahoo.fr ashbaughm@missouri.edu amel.atallah@fst.rnu.tn favhadiev@ksu.ru abdelhalim.azzouz.cus@gmail.com bahrisidimohamed@yahoo.fr c.bandle@gmx.ch mashhour\_ibrahim@yahoo.com banuelos@math.purdue.edu sbaraket@ksu.edu.sa nedra.belhajrhouma@fst.rnu.tn lamiatoukabri@vahoo.fr email Ali.Benamor@ipeit.rnu.tn rafael.benguria@googlemail.com benguriadepa@wisc.edu mrhouma@hotmail.com benmoussageo@gmail.com email malaklak2006@hotmail.com benomraneines@gmail.com soniabenothmane@fsb.rnu.tn chiara.bianchini@math.unifi.it habib.bouchriha@fst.rnu.tn b\_hakima2000@yahoo.fr saleh\_boulaares@yahoo.fr almut@math.toronto.edu fabian.cadiz@polytechnique.edu antonioc@us.es t.carroll@ucc.ie lchasman@knox.edu s.chemikh@yahoo.fr rym.chemmam@fst.rnu.tn anisa23in@gmail.com Hichem.Chtioui@fss.rnu.tn cianchi@unifi.it Bruno.Colbois@unine.ch makkia.dammak@gmail.com yohann.decastro@math.univ-toulouse.fr semrademirel@web.de James.Dilts@gmail.com bodo.dittmar@mathematik.uni-halle.de mdorff@math.byu.edu beccadorff@gmail.com ikmar.douchich@gmail.com dridi\_br@yahoo.fr Olivier.Druet@umpa.ens-lvon.fr eljazisalem@gmail.com elsoufi@univ-tours.fr mh.fantar@utm.rnu.tn Ali.Fardoun@univ-brest.fr ferone@unina it jfleckinger@gmail.com luigi.fontana@unimib.it rlfrank@math.princeton.edu friedlan@math.arizona.edu ngamara7@gmail.com leander.geisinger@mathematik.uni-stuttgart.de ghomi@math.gatech.edu normaliti@gmail.com alex.girouard@gmail.com nabil.gmati@ipein.rnu.tn habiba.guemri@gmail.com Mohamed.Hajjej@math.univ-bpclermont.fr mohamedalihamza@vahoo.fr

Harrell Evans Hasnaoui Abdelhalim Henrot Antoine Hermi Lotfi Hoffmann-Ostenhof Thomas Horrigue Samah Hupp Philipp Hynd Ryan Imsatfia Moheddine Iversen Mette Jbilou Asma Jedidi Wissem Jleli Mohamed Karoui Abderrazek Khenissi Moez Lambolev Jimmy Laptev Åri Laugesen Richard Lawlor Gary Lazhari M. Nejib Maagli Habib Maatoug Lamia Mahmoudi Fethi Majdoub Mohamed Maltsev Anna Moakher Maher Mondino Andrea Morgan Frank Morpurgo Carlo Music Michael Odzijewicz Tatiana Pacard Frank Pascu Mihai Philippin Gérard Ratzkin Jesse Rébaï Yomna Riahi Moncef Saada Ali Sahli Belkacem Salakhudinov Rustem Sampson Donald Selmi Abdelbaki Shafigullin Ilnar Sifi Mohamed Sing-Long Carlos Siudeja Bartlomiej Solórzano Pedro Tayachi Slim Tkitek Mahdi Torki-Hamza Nabila Trimeche Khalifa Tukhvatullina Alina Usman Muhammad Vernadakis Efstratios Vial Francisco Wei Na Zarati Said Zeglaoui Ahmed Zennir Khaled Zeraï Mourad Zorgati Hamdi Zribi Malek

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USA

harrell@math.gatech.edu hasnaoui.abdelim9@gmail.com Antoine.Henrot@mines.inpl-nancy.fr hermi@math.arizona.edu thoffman@esi.ac.at horrigues@yahoo.fr philipp@huppweb.de ryanhynd@math.berkeley.edu imsatfia@math.jussieu.fr mette.iversen@bristol.ac.uk Asma.Jbilou@unice.fr wissem\_jedidi@yahoo.fr jleli@ksu.edu.sa Abderrazek.Karoui@fsb.rnu.tn moez.khenissi@fsg.rnu.tn jimmy.lamboley@ceremade.da unhine fr a.laptev@imperial.ac.uk laugesen@illinois.edu lawlor@mathed.byu.edu mednejib.lazhari@mes.rnu.tn habib.maagli@fst.rnu.tn Lamia.Maatoug@fst.rnu.tn fethi.mah@gmail.com mohamed.majdoub@fst.rnu.tn annavmaltsev@gmail.com maher.moakher@enit.rnu.tn mondino@sissa.it Frank.Morgan@williams.edu morpurgoc@missouri.edu estill.music@gmail.com tatianao@ua.pt pacard@univ-paris12.fr mihai.pascu@unitbv.ro Gerard.Philippin@mat.ulaval.ca j.ratzkin@ucc.ie yomna.rebai@gmail.com moncef.ipeib@gmail.com Ali.Saada@ipein.rnu.tn sahlib@yahoo.com Rustem.Salahudinov@ksu.ru sampson.dcs@gmail.com Abdelbaki.Selmi@fsb.rnu.tn Shafigullin.IK@gmail.com mohamed.sifi@fst.rnu.tn casinglo@gmail.com siudeja@illinois.edu pedro@math.sunysb.edu slimtavachi@vahoo.fr mahdi.tekitek@math.u-psud.fr natorki@yahoo.fr khlifa.trimeche@fst.rnu.tn Alina.Tukhvatullina@ksu.ru m.usman08@imperial.ac.uk stratos@ugr.es fvialp@gmail.com nawei8382@gmail.com said.zarati@fst.rnu.tn ahmed.zeglaoui@gmail.com khaledzennir2@yahoo.com mourad.zerai@gmail.com hamdizorgati@yahoo.fr malek.zribi@insat.rnu.tn



Institut préparatoire aux études d'ingénieur de Bizerte

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For more information, contact vincent.rivasseau@nexteinstein.org



