ISOPERIMETRICAL PROBLEMS

The Eclectic Review, 1811

Art. II. ***A Treatise on Isoperimetrical Problems, and the Calculus of Variations.*** **By Robert Woodhouse, A. M. F. R. S. Fellow of Caius College, Cambridge. 8vo. pp. x. 154. Price 6s. Cambridge, Deighton. Black, Parry, and Kingsbury. 1810.**

AMONG the various circumstances which have tended to give a fresh impulse to the abstract sciences, at any point of their course, none seems to have been more beneficial, than the practice, which has prevailed more or less among mathematicians for the last two centuries, of proposing problems to exercise the invention and call forth the skill and ingenuity of each other. This has been remarkably exemplified in the history of the modern or infinitesimal calculus. We owe its first invention to the genius of Newton; but most of its subsequent improvements and modifications have been the result of a gradual expansion of views, occasioned by the solution of some problem or problems, the proposers of which had scarcely any conception of the amazing augmentations to science which the complete solution of their own questions would produce.

The first problem that related to a species of maxima and minima different from those usually treated by the fluxionary calculus, was proposed by Newton himself in the Principia, being that concerning the solid of least resistance. But the peculiar doctrine on which this and various kindred problems depended was brought into discussion, in consequence of John Bernoulli's proposing, in the Leipsic Acts for June 1696, the determination of the *curve of quickest descent.* The solution of this problem surpassed the genius of Leibnitz; who, nevertheless, according to his usual custom, *intimated* that he had solved it, but begged that John Bernoulli would lengthen the period for receiving solutions, (generous man!) that other philosophers might strip him of the honour of first solving it. Accordingly, Newton gave, without the authority of his name, the first public solution in the Leipsic Acts for May 1697: 'Quoique* l'auteur de cette construction par un excés de 'modestie ne se nomme pas, nous savons pourtant indubita-'blemente, par plusieurs circonstances, que c'est le celebre 'Newton, et quand meme nous ne le saurions point d'ailleurs, 'ce seroit assez de le connoitre par un echantillon, comme *ex* '*ungue Leonem.*' This simple and elegant construction is given at p. 232. Vol. I. Gregory's Mechanics: but that author seems to be mistaken, in supposing that it was *first* published in the Philosophical Transactions, No. 224.

The next problem in the series of developement, was exhibited in James Bernoulli's famous Programma of 1697, in the following terms: 'Quæritur ex omnibus isoperimetris, super 'communi basi *BN* constitutis, illa *BFN*, que non ipsa 'quidem maximum comprehendat spatium, sed faciat, ut 'aliud curva *BZN* comprehensum sit maximum, cujus appli-'cata *PZ* ponitur esse in ratione quavis multiplicata, vel sub-'multiplicata, rectæ *PF*, vel arcus *BF*, hoc est, quæ sit 'quotacunque proportionalis ad datam *Ag* rectam *PF*, cur-'vamve *BF*.' This problem and its dependent inquiries became the apple of discord between the two learned brothers, John and James Bernoulli, occasioning not merely a controversy but a quarrel between them, which only terminated with the death of the latter. Indeed John gave ample proofs *sixteen* years after his brother's death, that he had not yet forgiven him; though, as Mr. Woodhouse remarks, 'that event, the lapse of time, the recollection of his brother's kindness, a zeal for a brother's fame, ought to have assuaged and laid to sleep all angry passions.' James Bernoulli's original solution was upon correct principles, though it admitted of improvement in point of brevity and perspicuity; while John's was really defective, as his brother had uniformly maintained. John Bernoulli considered only two elements of the curve; whereas it is requisite to employ *three*, or to introduce some equivalent condition. In problems such as that relating to

* Joann. Bernoulli, Opera, tom. i. p. 197.

the curve of quickest descent, where it is simply required to fulfil the condition of the maximum or minimum, the applying of this condition to two elements of the curve is sufficient to determine its differential equation. But when, in addition to the maximum or minimum, the curve must possess a farther property, namely, that of being isoperimetrical to another, this new condition requires, that a third element of the curve shall have a certain inclination with respect to the other two: and every determination founded merely on the first condition, will exhibit false results; except in those cases where a curve cannot satisfy any of the two conditions, without fulfilling the other at the same time. This essential condition of the *three* elements was introduced by John Bernoulli so late as 1718; and even then he had not candour enough to acknowledge, that his new solution was in substance the same as his brother's, though given in a form which considerably abridges the computation.

The consideration of this and some kindred problems, in the hands of Euler and Lagrange, led to the discovery of the Calculus of *Variations*—the other subject of Mr. Woodhouse's book. In this calculus, having given an expression or function of two or more variable quantities, of which the relation is expressed by a determinate law, we can find what that function will become, when the law itself is supposed to experience any indefinitely small variation, occasioned by the variation of one or more of the terms which express it. This calculus furnishes almost the only means of resolving a multitude of problems, *de maximis et minimis*, whose difficulty is very far greater than in problems usually referred to the fluxionary or differential calculus. Such, for example, is the problem which requires the curve that will conduct a body falling, in virtue of its acceleration, to any given point, or right or curved line, in the shortest time. In general, every problem of this nature is reduced to the finding the maximum or the minimum of a differential formula, such as $\int Z\,dx$, where Z is a function of x, or of constant quantities, or of x and y, or of x, y, z, and even still more variable quantities: indeed Z may contain integrals, as $\int V$, or integrals of integrals, as $\int V \int v$, &c. and it is the manner of taking the variation of these expressions which is prescribed by the rules of this calculus.

Of this calculus, M. Lagrange is the true inventor: yet the two Bernoullis in solving the problem of which we have already spoken, and M. Euler, effected much in preparing the way for Lagrange. Euler's learned work—"Methodus Inveniendi lineas curvas proprietate maximi minimive gaudentes," &c. (1774) which is a wonderful specimen of profundity and sagacity, contains essentially all the requisite methods of solution, and is enriched with a great variety of examples and illustrations. Yet it wants a new algorithm, a compendious method by which the theorems may be established without ambiguity and circumlocution, and an obvious principle of application to the several cases which should arise. These were supplied by the fertile genius of Lagrange, partly in the second volume of the Turin Memoirs (A. D. 1762), and more completely in the fourth volume of those Memoirs (A. D. 1767), and in his "Theorie des Fonctions Analytiques' (1797), and his 'Leçons sur le Calcul des Fonctions' (1806). A very perspicuous, though brief sketch of this theory is also given by M. Bossut in his 'Calcul Integral;' another by Lacroix in his admired performance on the same subject; and another (which several foreign mathematicians prefer to either Bossut's or Lacroix's) by M. Cousin.

From this concise history of the methods of Isoperimeters and Variations, it will be seen that no English author has attempted a treatise upon them. Some particular problems have, it is true, been considered by Maclaurin, Simpson, Emerson, and a few others; but none of these mathematicians seems to have confined his attention long enough to this interesting branch of investigation, to strike out a general theory applicable to the several cases that might occur. Simpson's seventh tract does not furnish a complete exception to this remark: for, besides that the rule he investigates applies only to isoperimetrical problems, he does not follow his own rule in some of the examples he has given. Mr. Woodhouse's, then, is the *first* treatise on these subjects which has yet been given in the English language, and the second *distinct* treatise which has been offered in any language—the first being Euler's *Methodus Inveniendi,* &c. before mentioned. Our author's reasons for undertaking the present work, as well as an account of the plan he pursues in it, will appear by the following quotation from his preface.

'When Lagrange, in 1760, published his new method of solving problems of maxima and minima, he composed his memoir for mathematicians, familiar with its subject, and well versed in the researches of the Bernoullis and of Euler. Accordingly, he very briefly states the principles of his calculus, and enters into no explanation on the nature of the subject. His compendious method of computation, however, has been adopted; and subsequent authors have composed their treatises very much on the plan of Lagrange's memoir, with some, but slight and imperfect, preliminary explanation. These treatises, however, the student is expected to understand; that is, if the matter be fairly stated, he is expected to understand an intricate subject, with advantages much less than consummate mathematicians before him enjoyed; since there is neither proper explanation presented to him, nor is he directed, by way of preparation, previously to consult the works of Euler and the Bernoullis.

'Such are, in my opinion, the defects of existing methods. Still, however, I have not composed a treatise on the subject, by merely remedying them; that is, by inserting formulæ of sufficient extent, and by more fully explaining and illustrating their principles. But, on a novel plan, I have combined the historical progress with the scientific developement of the subject; and endeavoured to lay down and inculcate the principles of the calculus, whilst I traced its gradual and successive improvements.

'If this has been effected, which I think it has, in a compass not very wide of that which a strictly scientific treatise would have required, the only serious objection against the present plan is, in part, obviated. For, there is little doubt, the student's curiosity and attention will be more excited and sustained, when he finds history blended with science, and the demonstration of formulæ accompanied with the object and the causes of their invention, than by a mere analytical exposition of the principles of the subject.' pp. iii, iv.

Conformably with the plan Mr. Woodhouse has thus prescribed himself, he divides his work into eight chapters; the principal subjects of which will appear from the subjoined analysis.—Chapter the 1st. relates to the problem of the curve of quickest descent, and contains a full developement of the principle of John Bernoulli's solution. In the 2nd chapter Mr. Woodhouse announces the isoperimetrical problems proposed by James Bernoulli; describes the nature of the solution given by John Bernoulli; explains the distinction between his fundamental and specific equations; shews the application of them to the curve of quickest descent, and of a given length; describes Brook Taylor's solution of isoperimetrical problems; and points out the imperfections of the methods employed by him and the Bernoullis. The 3d chapter contains an account of Euler's first memoir on isoperimetrical problems, and of his very ingenious table of formulæ, with their application to the solution of some problems: it also contains a brief account of the methods of Maclaurin, Emerson, and Simpson, and points out their restrictions. The 4th chapter is employed in describing Euler's second memoir, (Comm. Petrop. tom. viii) his *general* formulæ of solution in that memoir, in tracing the characters of distinction between different problems, and in pointing out exceptions to the general formulæ: Mr. W. here shews in what manner the *class* of problems leads to the determination of the number of ordinates that must vary, and the *order*, the number that must be introduced into the computation. Chapter the 5th is devoted principally to Euler's tract, intitled "Methodus Inveniendi Lineas curvas," &c: in this the author explains the distribution of cases into absolute and relative maxima and minima, exhibits rules for finding the increment of quantities dependent on their varied state, and more valuable formulæ of solution. The 6th chapter relates to Lagrange's first memoir on the theory of variations: and here Mr. Woodhouse shews the uses of an appropriate symbol, such as δ, to denote the variation of a quantity, traces the similarity between the differential calculus and that of variations, and deduces the principal rules for finding the variation in any proposed case; a new process is also exhibited of deducing Euler's formulæ, and several new formulæ are given, with their applications to some problems. In the 7th chapter Lagrange's general method of treating isoperimetrical problems is explained, and especially the nature and use of the equation of limits; and several useful remarks are added to shew the method of reducing cases of relative maxima and minima to those of absolute. In the 8th and last chapter, the author has first shewn how to deduce several subordinate formulæ from Euler's general formulæ; being such as, though they are more limited, materially expedite the solution of problems. He then presents a collection of thirty problems with their solutions. Of these some are very curious and interesting, especially those relating to the inquiry of the brachystochrone in all its varieties.

From a work like the present, in which almost every page is so intimately connected with what precedes it, (either by the peculiarities of the notation, or the enchainment of logical method,) as scarcely to admit of any separation from it without becoming unintelligible, it is difficult to make any quotations. Perhaps, however, the following extracts may serve to communicate to the scientific reader, as well the spirit of the methods to which they relate, as the manner in which our author treats the respective subjects.

‘Euler reduced isoperimetrical problems of the second class to a dependance on two similar equations of the form

$$P\,bg - (P + dP)\,ci = 0,$$

the determination of P depending on the proposed properties: for, if either the isoperimetrical property, or that from the maximum were $\int T.dx$, $T = f(y)$, P would equal $\frac{dT}{dy}$, or $\frac{dT}{dy}.dx$. If the property were $\int T.dy$, $T = f(x)$, P would $= \frac{dT}{dx}$ or $\frac{dT}{dx}.dx$. If the property were $\int T.ds$, $T = f(x)$, P would equal $d\left(T.\frac{dy}{ds}\right)$; and by observation on the resulting forms for P, Euler generalised his conclusions, and arranged them in a table, after the manner of the subjoined specimen.*

Proprietates propositæ.		Valores Litteræ P respondentes.
I. $\int T.dx$ . . .	$dT = M dy$† . .	$P = M.dx$

‘* *Comm. Acad. Petrop.* tom. VI. p. 141.

‘† $dT = M\,dy$, and $M = \frac{dT}{dy}$, or M is the differential coefficient of T, making

II. $\int T.dy$ . . . $dT = Ndx$ . . . $P = N.dx$

III. $\int T.ds$ . . . $dT = Ndx$ . . . $P = d.\left(T.\frac{dy}{ds}\right)$

IV. $\int T.ds$ . . . $dT = Mdy$ . . . $P = d.\left(T\frac{dy}{ds}\right) - Mds$,

&c.

and of these forms he gave fifteen, by reference to which, any problem belonging to the second class might be solved.

'For instance, suppose the curve to be required, which, amongst all others of the same length, should contain the greatest area. Here,

the maximum property $B, = \int y\,dx$,

the isoperimetrical $A, = \int ds = a.$

By Form I. $T = y$; $\therefore M = 1$, $P = dx$.

By Form III. $T = 1$; $P = d\left(\frac{dy}{ds}\right)$ or $R = d\left(\frac{dy}{ds}\right)$

Hence, since the equation is $P \pm aR = 0\ dx \pm a.d\left(\frac{dy}{ds}\right) = 0$, or

$x + a\frac{dy}{ds} - c = 0$ ($c =$ correction); and by reduction,

$$dy = \frac{(x-c).dx}{\sqrt{(a^2 - [x-c]^2)}}$$ an equation to a circle.*

'Again, suppose the curve to be required, which, amongst all others of the same length, shall, by a rotation round its axis, generate the greatest solid. Here,

$B = \int y^2.dx$; $\therefore$ by Form I. $T = y^2$, $M = 2y$, $P = 2y.dx$

$A = \int ds$; $\therefore$ by Form III. P or $R = d\left(\frac{dy}{ds}\right)$;

Hence, $2y.dx + a.d\left(\frac{dy}{ds}\right) = 0$,

or, $2y.dx + a.\frac{ds.d^2y - dy.d^2s}{ds^2} = 0.$

But since dx is constant, and $ds^2 = dx^2 + dy^2$, $ds.d^2s = dy.d^2y$; therefore, substituting,

$$2y.dx + a.\frac{d^2y.dx^2}{ds^3} = 0,$$

and, $2y + \frac{a.d^2y.dx}{(dx^2+dy^2)^{\frac{3}{2}}} = 0$; multiply by dy, and integrate, and we have

---

in T, y to vary. Similarly, $N = \frac{dT}{dx}$ is the differential coefficient making in T, x to vary. If T should contain both x and y, that is, if $dT = Mdy + Ndx$, then M and N would become partial differential coefficients. See *Princ. Anal. Calc.* p. 79.

* See Emerson's *Fluxions*, third edition, p. 187; also Simpson's *Fluxions*, p. 485.

$$y^2 - \frac{a \cdot ds}{(dx^2 + dy^2)^{\frac{1}{2}}} = c \text{ [} c \text{ correction,]}$$

whence, $ds = \dfrac{(y^2 - c)\, dy}{\sqrt{(a^2 - [y^2 - c]^2)}}$, an equation to the elastic curve; and which in a particular case, when $c = 0$, becomes

$$dx = \frac{y^2 dy}{\sqrt{(a^2 - y^4)}},$$

and the curve in this case is called the rectangular elastic curve.*

'As a third example, let the curve be required, which, amongst all others of the same length, shall have its center of gravity most remote from the axis. Here, (calling x the distance from the axis) $B = \int \frac{x.ds}{s}$;

$\therefore$ by Form III. [since s is a given quantity] $P = d\left(x\frac{dy}{ds}\right)$ again $A = \int ds$; $\therefore$ by Form III. P, or $R = d\left(\frac{dy}{ds}\right)$

$$\text{Hence, } a \cdot d\left(\frac{dy}{ds}\right) + d\left(x\frac{dy}{ds}\right) = 0;$$

$$\therefore (a + x)\frac{dy}{ds} = c, \text{ or } c\,ds = (a + x).dy$$

an equation to the catenary.

'This example could not have been solved by Euler's table, if the property had been any other than the isoperimetrical: for s, an integral, $= \int dx \sqrt{\left(1 + \frac{dy^2}{dx^2}\right)}$; and Euler gives, in this memoir, no general method of finding the resulting equation, such as P is in his table, when the analytical expression of a property involves integrals. See tom. VI. p 144.

'By means of this table, the practical solution of isoperimetrical problems, was, as it has been already said, very materially expedited. In a subsequent part of his memoir,† Euler increases his table by nine new forms: making the whole number twenty-four. And although this table is now superseded, yet its examination is not without interest, since we may discover in it the parcels of that general formula, which the author afterwards exhibited.' pp. 40—44.

The subjoined quotation serves to explain an essential part of Lagrange's method, and is so simple as to need no particular explanation.

'Whatever be the function V,

if $dV = Mdx + Ndy + Pdp + Qdq +$ &c.

then $\delta V = M\delta x + N\delta y + P\delta p + Q\delta q +$ &c.

'Since the processes for finding the differential and variation differ only in the symbols dy, δy, which are arbitrary; it is plain, if both operations are to be performed on an analytical expression, that it is matter of indif-

* See Simpson's *Fluxions*, p. 486. where the solution is not general.

† Comm. Petrop. tom. vi. p. 146.

ference, which operation is performed first: or, if the symbols *d*, , meet together denoting operations, we may, at our pleasure, change their order: for instance $d\delta y$ and δdy are alike significant; for $d\delta y$ means the firs term of two successive values of δy, or $= \delta\,(y + dy) - \delta y = \delta y + \delta dy - \delta y = \delta dy$: again, if V for instance be a function of y; then

$$dV = \frac{dV}{dy}\, dy, \quad \text{and } \delta dV = \frac{d^2V}{dy^2}.\,\delta y.dy$$

$$\delta V = \frac{dV}{dy}.\,\delta y, \quad \text{and } d\delta V = \frac{d^2V}{dy^2}.\, dy.\delta y;$$

$$\therefore \delta dV = d\delta V,$$

or, in a particular instance, when $V = y^n$,

$$d\,(y^n) = 1^{st} \text{ term of } [(y+dy)^n - y^n] = ny^{n-1}.dy$$

$$\delta d(y^n) = 1^{st} \text{ term of } ndy \times [(y+\delta y)^{n-1} - y^{n-1}] = n\,(n-1)\,y^{n-2}.dy.\delta y,$$

$$\delta(y^n) = 1^{st} \text{ term of } [(y+\delta y)^n - y^n] = ny^{n-1}.\delta y$$

$$d\delta(y^n) = 1^{st} \text{ term of } n\delta y \times [(y+dy)^{n-1} - y^{n-1}] = n\,(n-1)\,y^{n-2}.\delta y\, dy;$$

$$\therefore \delta d\,(y^n) = d\delta\,(y^n).$$

And, by similar processes, $d^2\delta V = \delta d^2V = \delta ddV = d\delta dV$

$$d^3\delta V = \delta d^3V = d\delta d^2V = d^2\delta dV.$$

'This rule is, in Lagrange's method, of the greatest importance; it is an essential part of it. Amongst other uses, it enables us when an integral is concerned, to introduce the symbol δ within the symbol ($\int$) of the integral: thus, since the symbols d and $\int$ indicate reverse operations,

$$V = d\int V; \;\therefore \delta V = \delta d\int V = d\delta\int V.$$

Hence, taking the integrals on each side

$$\int\delta V = \int d\delta\int V = \delta\int V \;.\;.\;.\; [a]$$

This result may be easily extended to double and treble integrals: for if $V = \int W$, then $\delta V = \delta\int W = \int\delta W$ by [a]; $\therefore \int\delta V = \int\int\delta W$: but $\int\delta V = \delta\int V = \delta\int\int W$, consequently $\delta\int\int W = \int\int\delta W$.' pp. 83—85.

It is now time for us to characterize this work, which we may do very shortly, by saying that we prefer it very much to any preceding performance of the same author. It is more methodical, more perspicuous, infinitely less affected, and will, we doubt not, be far more useful. In a few instances the links which connect one method with another in the history of discoveries, are not all supplied; and two or three inadvertencies have escaped the author. But the chief things of which *young* mathematicians will complain, after they have read this treatise, will be ambiguities arising from the defects of the system of notation pursued by foreigners, and adopted, *con amore*, by Mr. Woodhouse. Thus in some cases, d, δ, mark the extremities of a line which is a variation of an ordinate, while, in others they are employed to designate, the former the *differential*, the latter the *variation* of a quantity;—and then the reader needs to be told, (as at the note, p. 45.) that '$d\,\delta$ has no connexion whatever with the separate symbols d, δ.' In some parts of the work, a letter, P for example, stands for a function of x; in others, the same letter is used simply as a coefficient: We may perhaps be told again, in a note, that P here is 'merely a *coefficient*, and different from the P of the preceding chapter;' but when a reader dips into this work for occasional reference, is it to be wished that he should be compelled to hunt out these notes, before he can tell in what light to contemplate the symbols he meets with? Some of these ambiguities in notation are mere inadvertencies in the author, and might have been avoided with a very little additional care and reflection; but others are inevitable consequences of the foreign notation, and can only be escaped by rejecting that notation altogether. Let us, however, attend to Mr. Woodhouse's reasons for adhering to it.

'In a former work,* I adopted the foreign notation, and the present occasion furnishes some proof of the propriety of that adoption. In the calculus of variations, it is necessary to have symbols denoting operations, similar to those that take place in the differential calculus: now, d being the symbol for the latter, δ is a most convenient one for the former: analogous to δ there is no symbol in the English system of notation. If then I had used the fluxionary notation with points or dots, I must have invented symbols corresponding to δ and the characters formed by means of it. But, the invention of merely new symbols is in itself an evil. M. Lagrange indeed, whose power over symbols is so unbounded that the possession of it seems to have made him capricious, has treated the subject of variations without the foreign notation; this he rejects altogether; and, which is strange, has employed the English notation, but not adopted its signification. Thus, with him, $\dot{x}$ is not the fluxion, but the variation of x; the fluxions or differentials of quantities are not expressed by him, but solely the fluxionary or differential coefficients: thus, if u be a function of x, $u' \left(= \frac{du}{dx} \text{ or } = \frac{\dot{u}}{\dot{x}}\right)$ is the differential coefficient. What advantages are to arise from these alterations it is not easy to perceive: yet they ought to be great, to balance the plain and palpable evils of a confusion in the signification of symbols, and of the invention of a system of notation to represent what already was represented with sufficient precision. No authority can *even* sanction so capricious an innovation.' Preface, pp. vi, vii.

Now, besides that there is a very forcible objection which presents itself to the mind of every mathematician when he *first* thinks or hears of prefixing one algebraical letter to another, the former to denote an operation performed upon the latter which represents a quantity,—there is no advantage that we can perceive in point of either facility or elegance, which

* Principles of Analytical Calculation, 1803.

the foreign possesses over the English notation. We are persuaded, indeed, that no unbiassed person can examine the 'table of the foreign and the corresponding English notation,' given by Mr. Woodhouse at the end of the preface of the work before us, without deciding in favour of the *English* method. In various inquiries where the fluxions of a connected series of quantities, *a*, *b*, *c*, *d*, *e*, &c. are to be employed, they are denoted with perfect freedom from ambiguity by the English notation; but how are they to be represented by the differential notation? Must the series be broken for the purpose of excluding the *d*? What then becomes of the universality of algebraic representation? And, what if the *d* should stand for density, or diameter, or distance, or any other subject whose initial is *d*,—must we lose the advantage of employing the initial, because the differential notation has monopolized the use of that letter? But, says Mr. Woodhouse, 'analogous to δ there is no symbol in the English system of notation. If I had used the fluxionary notation with points or dots, I must have invented symbols corresponding to δ, and the characters formed by means of it.' Well: and where would have been the difficulty of effecting this? Let the variation be denoted by a dot *below* the quantity, as the fluxion is uniformly represented by one *above* it; and, in that case, we fancy both *d* and δ may very safely be dispensed with, as representatives of operations. In that case,

$\delta d^2 V$ will in our notation be $(\ddot{V})$.

$d^2 \delta V$ be $(\underset{\cdot}{V})^{\cdot\cdot}$

and either of them will be equivalent to $\underset{\cdot}{\ddot{V}}$. We have put down these expressions solely to shew that the thing is not *impossible*, according to the English notation; and by no means intend to affirm that we have struck out the best method of accomplishing it. As to the conduct of Lagrange, which has called forth Mr. Woodhouse's animadversions, there can be but little doubt that he was forced into it by a conviction of the ambiguities and disadvantages attending the foreign notation; while he employed the *dot*, by way of experiment, to see whether it was not possible to triumph over the English philosopher, nearly a century after his death, by appropriating his notation to a purpose widely different from its original use, and obtaining currency to the new modification. Such are the arts by which foreigners try to cast the greatest of mathematical inventions into oblivion: First, they give the science a new name; then, they devise a new character to denote the specific operations; then, they hide the invention under the jargon of a new metaphysics; and, finally, they deprive the

English invention of its last distinguishing vestige, its notation, and appropriate it to another use! And yet, there are to be found two or three Englishmen, and five or six Scotchmen, who, notwithstanding all this, extol the *liberality*, as well as the talents of French mathematicians; and seem as utterly unconscious of the injury attempted to be done to their great countrymen, as are even the illustrious dead, on whose reputation foreigners thus trample, and whose imperishable memory they are thus labouring to extinguish!

THE

Eclectic Review.

VOL. VII. PART II.

FROM JULY, TO DECEMBER, 1811, *INCLUSIVE.*

Φιλοσοφιαν δε ου την Στωικην λεγω, ουδε την Πλατωνικην, η την Επικουρειον τε και Αριστοτελικην· αλλ᾽ ὁσα ειρηται παρ᾽ ἑκαστη των ἁιρεσεων τουτων καλως δικαιοσυνην μετα ευσεβους επιστημης εκδιδασκοντα, τουτο συμπαν το ΕΚΛΕΚΤΙΚΟΝ φιλοσοφιαν φημι. Clem. Alex. *Strom. Lib.* 1.

LONDON:

Printed for LONGMAN, HURST, REES, ORME, AND BROWN, PATERNOSTER-ROW.

1811.