

## Problem Solutions set2

**Problem 1.36.** (Pumping up a bicycle tire.)

(a) From equation 1.40 (derived in the previous problem), we have

$$P_f V_f^\gamma = P_i V_i^\gamma, \quad \text{or} \quad V_f = V_i \cdot \left( \frac{P_i}{P_f} \right)^{1/\gamma} = (1 \text{ liter}) \left( \frac{1 \text{ atm}}{7 \text{ atm}} \right)^{5/7} = 0.25 \text{ liters},$$

where  $i$  and  $f$  refer to initial and final, and I've set  $\gamma = 7/5$  for air, which has five degrees of freedom per molecule at ordinary temperatures.

(b) The pressure as a function of volume is  $P = \text{constant}/V^\gamma$ , where the constant can be evaluated from the initial conditions as  $P_i V_i^\gamma$ . The work done is then

$$\begin{aligned} W &= - \int P dV = -P_i V_i^\gamma \int_{V_i}^{V_f} V^{-\gamma} dV = -P_i V_i^\gamma \left( \frac{V^{-\gamma+1}}{-\gamma+1} \right) \Big|_{V_i}^{V_f} \\ &= \frac{P_i V_i^\gamma}{\gamma-1} \left( \frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}} \right) = \frac{P_i V_i}{\gamma-1} \left[ \left( \frac{V_i}{V_f} \right)^{\gamma-1} - 1 \right] \\ &= \frac{1 \text{ liter} \cdot \text{atm}}{2/5} [(4.0)^{2/5} - 1] = 1.86 \text{ liter} \cdot \text{atm} = 188 \text{ J}. \end{aligned}$$

(c) From equation 1.39 we can write  $V_f T_f^{f/2} = V_i T_i^{f/2}$ , or

$$T_f = T_i \cdot \left( \frac{V_i}{V_f} \right)^{2/f} = (300 \text{ K}) \left( \frac{1 \text{ liter}}{0.25 \text{ liters}} \right)^{2/5} = (300 \text{ K})(1.74) \approx 520 \text{ K}.$$

**Problem 1.37.** From equation 1.39 we can write  $V_f T_f^{f/2} = V_i T_i^{f/2}$ , where  $i$  represents the initial state and  $f$  represents the final state. I'll assume an initial temperature of 300 K and  $f = 5$  for air. Solving for  $T_f$  then gives

$$T_f = T_i \cdot \left( \frac{V_i}{V_f} \right)^{2/f} = (300 \text{ K})(20)^{2/5} = (300 \text{ K})(3.31) \approx 1000 \text{ K}.$$

This is presumably hot enough to ignite the fuel as soon as it is injected, without the aid of a spark plug.

**Problem 1.39.** (Bulk modulus and speed of sound.)

(a) During an isothermal compression, according to the ideal gas law,

$$\frac{dP}{dV} = \frac{d}{dV} \frac{NkT}{V} = -\frac{NkT}{V^2},$$

and therefore the bulk modulus is

$$B_{\text{isothermal}} = -V \frac{dP}{dV} = \frac{NkT}{V} = P.$$

During an adiabatic compression, on the other hand, the pressure of an ideal gas obeys  $PV^\gamma = C$  for some constant  $C$ . This implies

$$\frac{dP}{dV} = \frac{d}{dV} \frac{C}{V^\gamma} = -\gamma \frac{C}{V^{\gamma+1}},$$

and therefore

$$B_{\text{adiabatic}} = -V \frac{dP}{dV} = \gamma \frac{C}{V^\gamma} = \gamma P.$$

Since  $\gamma$  is always greater than 1, the adiabatic bulk modulus is always larger than the isothermal bulk modulus. This makes sense, because it's harder to compress a gas adiabatically (as its temperature rises) than isothermally.

- (b) Sound typically travels *much* faster than heat, so there's no way that heat could flow back and forth between the compressed and rarified portions of sound wave fast enough to accomplish any temperature equilibration. Therefore we can treat these compressions as approximately adiabatic, with no significant heat flow at all.
- (c) Let  $m$  be the average mass of a gas molecule. Then the density,  $\rho$ , is  $Nm/V = mP/kT$  by the ideal gas law. The speed of sound is therefore

$$c_s = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{mP/kT}} = \sqrt{\frac{\gamma kT}{m}} = \sqrt{\frac{\gamma RT}{M}},$$

where  $M$  is the mass of a mole of the gas. This result is the same as equation 1.21 for the rms speed of the gas molecules, except that the 3 is here replaced by  $\gamma$ . In other words, the speed of sound is slower than the rms molecular speed, by a factor of  $\sqrt{3/\gamma}$ . (Note that  $\gamma$  is always between 1 and 5/3, being largest for a monatomic gas.) This result makes sense: The sound wave is carried by the molecules, so its speed should be comparable to the average molecular speed, and definitely no faster. Numerically, for air at room temperature,

$$c_s = \sqrt{\frac{1.4(8.31 \text{ J/K})(300 \text{ K})}{0.029 \text{ kg}}} = 347 \text{ m/s}.$$

- (d) At high altitude, the pressure of the atmosphere is less than at sea level. But the result of part (c) shows that for an ideal gas, the speed of sound is independent of pressure—it depends only on temperature. (Both  $B$  and  $\rho$  are proportional to pressure, but this dependence cancels in the ratio  $B/\rho$ .) If the environment throws your bagpipes out of tune, it must be the temperature, not the lower pressure, that causes this effect. In fact, the concert referred to was in a heated auditorium so there should have been no significant effect due to temperature either. In any case, the musician managed to tune his pipes just fine and the performance was superb.

**Problem 1.41.** (Measuring a heat capacity.)

- (a) The heat gained by the water is

$$m_w c_w (\Delta T)_w = (250 \text{ g})(4.186 \text{ J/g}\cdot^\circ\text{C})(4^\circ\text{C}) = 4186 \text{ J}.$$

- (b) The heat lost by the metal must be the same as the heat lost by the water, 4186 J, since there are no other energy transfers going on and energy must be conserved.  
 (c) The heat capacity of the chunk of metal must therefore be

$$C_m = \frac{Q}{\Delta T_m} = \frac{-4186 \text{ J}}{-76^\circ\text{C}} = 55 \text{ J}/^\circ\text{C}.$$

- (d) The specific heat capacity is the heat capacity per unit mass,

$$c_m = \frac{C_m}{m_m} = \frac{55 \text{ J}/^\circ\text{C}}{100 \text{ g}} = 0.55 \text{ J/g}\cdot^\circ\text{C}.$$

**Problem 1.42.** The heat lost by the water should be approximately equal to the heat gained by the pasta. Therefore,

$$m_w c_w (T_w - T_f) = m_p c_p (T_f - T_p),$$

where  $w$  stands for water,  $p$  for pasta, and  $f$  for the common final temperature. Solving for  $T_f$  gives

$$\begin{aligned} T_f &= \frac{m_w c_w T_w + m_p c_p T_p}{m_w c_w + m_p c_p} \\ &= \frac{(1500 \text{ g})(4.186 \text{ J/g}\cdot^\circ\text{C})(100^\circ\text{C}) + (340 \text{ g})(1.8 \text{ J/g}\cdot^\circ\text{C})(25^\circ\text{C})}{(1500 \text{ g})(4.186 \text{ J/g}\cdot^\circ\text{C}) + (340 \text{ g})(1.8 \text{ J/g}\cdot^\circ\text{C})} = 93.3^\circ\text{C}. \end{aligned}$$

So we would expect the water temperature to drop by nearly  $7^\circ\text{C}$ , assuming that equilibration happens faster than the stove can provide any significant additional heat. To prevent the water temperature from dropping so much, it might be wise to use a bigger pot of water.

**Problem 1.47.** The amount of heat that I want to remove from the tea is

$$Q = (200 \text{ g})(1 \text{ cal/g}\cdot^{\circ}\text{C})(35^{\circ}\text{C}) = 7000 \text{ cal}.$$

This heat enters the ice, brings it up to  $0^{\circ}\text{C}$ , melts it, and then brings the melted water up to  $65^{\circ}\text{C}$ . If the mass of the ice is  $m$ , then

$$Q = m(0.5 \text{ cal/g}\cdot^{\circ}\text{C})(15^{\circ}\text{C}) + m(80 \text{ cal/g}) + m(1 \text{ cal/g}\cdot^{\circ}\text{C})(65^{\circ}\text{C}).$$

Solving for  $m$  gives  $m = 46 \text{ g}$ .

**Problem 1.48.** Consider a  $1\text{-m}^2$  patch of snow-covered ground, which is covered by  $1 \text{ m}^3$  of ice (and an equal amount of air mixed with it, which we can neglect). That's nearly 1000 kg, so if it's already at  $0^{\circ}\text{C}$ , the energy needed to melt it is 80 million calories or 335 MJ. Now direct sunlight provides to this patch of ground 1000 J/s, but only 10%, or 100 J/s, is absorbed. A full day of sunshine in late spring might be equivalent to 8 hours, or about 30,000 seconds, of direct sunlight. So in one day the snow absorbs about 3 MJ of solar energy. That means it should take more than 100 days, or 14 weeks, for the snow to melt! Probably, though, the snow absorbs more energy from other sources such as the air and any nearby rocks or trees.

**Problem 1.50.** (Combustion of methane.)

- (a) From the table on page 404, line 8, the change in enthalpy upon *forming* one mole of methane from elemental carbon and hydrogen is  $-74.81 \text{ kJ}$ ; the change in enthalpy for the reverse reaction is therefore  $+74.81 \text{ kJ}$ .
- (b) From the same table,  $\Delta H$  to form one mole of  $\text{CO}_2$  is  $-393.51 \text{ kJ}$  (line 14), while  $\Delta H$  to form two moles of  $\text{H}_2\text{O}$  vapor is  $2 \times (-241.82 \text{ kJ}) = -483.64 \text{ kJ}$  (page 405, line 5).
- (c) Imagine first converting the mole of methane into elemental carbon and hydrogen, then taking these elements and some oxygen and converting them into  $\text{CO}_2$  and  $\text{H}_2\text{O}$ . The total change in enthalpy for all these steps is

$$\Delta H = +74.81 \text{ kJ} + (-393.51 \text{ kJ}) + (-483.64 \text{ kJ}) = -802.34 \text{ kJ}.$$

Now the actual reaction doesn't occur by this sequence of steps, but the end result is the same, so the net change in enthalpy must be the same: a decrease of 802 kJ.

- (d) In a constant-pressure process with no other work done, the heat given off is precisely equal to the decrease in enthalpy, in this case, 802 kJ.

- (e) In a constant-pressure process,  $\Delta U = \Delta H - P \Delta V$ . This reaction starts with three moles of gas and ends with three moles of gas, so  $\Delta V$  is almost exactly zero (by the ideal gas law). Therefore  $\Delta U$  is the same as  $\Delta H$ ,  $-802$  kJ. However, if the  $\text{H}_2\text{O}$  ends up as a liquid, things are a bit more interesting. Then  $\Delta H$  to form the  $\text{H}_2\text{O}$  from its elemental constituents is instead  $2 \times (-285.83 \text{ kJ}) = -571.66 \text{ kJ}$  (page 405, line 4), and the computation in part (c) yields a net  $\Delta H$  of  $-890.36 \text{ kJ}$ . But the volume of the system decreases substantially, because it starts with three moles of gas and ends up with only one (while the volume of the liquid is negligible):

$$\Delta V = \frac{RT}{P} \Delta n = -\frac{2RT}{P}.$$

The change in the system's energy is therefore

$$\begin{aligned} \Delta U &= \Delta H - P \Delta V = \Delta H + 2RT = (-890.36 \text{ kJ}) + 2(8.315 \text{ J/K})(298 \text{ K}) \\ &= -890.36 \text{ kJ} + 4.96 \text{ kJ} = -885.40 \text{ kJ}. \end{aligned}$$

The heat given off is a bit *more* than the amount of energy lost by the system; the difference comes from work done by the atmosphere as it compresses the system to a smaller volume.

- (f) A mole of methane has a mass of only 16 g, but two moles of  $\text{O}_2$  have a total mass of 64 g, giving a total of 80 g = 0.080 kg for the mass of fuel to produce the reaction given here. If the sun were composed entirely of such fuel in the correct proportions, it would contain

$$\frac{2 \times 10^{30}}{0.080} = 2.5 \times 10^{31}$$

moles of methane, which could give off a total of roughly

$$2.5 \times 10^{31} \times 800,000 \text{ J} = 2 \times 10^{37} \text{ J}$$

of heat, by the reaction considered here. But at a rate of about  $4 \times 10^{26}$  watts, this fuel would last only

$$\frac{2 \times 10^{37} \text{ J}}{4 \times 10^{26} \text{ J/s}} = 5 \times 10^{10} \text{ seconds},$$

or about 1600 years. Other chemical fuels might give somewhat longer lifetimes, but no chemical fuel could power the sun for more than a few thousand years.

**Problem 1.53.** On page 405 we see that the enthalpy of formation of a mole of atomic hydrogen is 217.97 kJ. To convert this enthalpy change to an *energy* change, we need to subtract off the  $P \Delta V$  term:

$$\begin{aligned}\Delta U &= \Delta H - P\Delta V = \Delta H - (\Delta n)RT \\ &= 217.97 \text{ kJ} - \frac{1}{2}(8.31 \text{ J/K})(298 \text{ K}) = 217.97 \text{ kJ} - 1.24 \text{ kJ} = 216.73 \text{ kJ}.\end{aligned}$$

(The 1.24-kJ difference between  $\Delta U$  and  $\Delta H$  is the work that you must do to make room for the additional half-mole of gas created during dissociation.) Now, to get the energy required to dissociate a single molecule, we divide by the number of molecules:

$$\epsilon_{\text{diss.}} = \frac{\Delta U}{N_{\text{H}_2}} = \frac{216.7 \text{ kJ}}{\frac{1}{2} \cdot 6.02 \times 10^{23}} = 7.20 \times 10^{-19} \text{ J} = 4.49 \text{ eV}.$$