HW-Solution-set5

4.1, 4.3, 4.4, 4.5, 4.11, 4.14

Problem 4.1. (Ideal gas engine with rectangular PV cycle.)

(a) The net work done by the gas during one cycle is

$$|W| = (P_2 - P_1)(V_2 - V_1) = (P_1)(2V_1) = 2P_1V_1,$$

while the heat absorbed (during steps A and B) is

$$Q_h = \frac{5}{2}V_1(P_2 - P_1) + \frac{7}{2}P_2(V_2 - V_1) = \frac{5}{2}V_1P_1 + 14P_1V_1 = \frac{33}{2}P_1V_1.$$

Therefore the efficiency is

$$e = \frac{|W|}{Q_h} = \frac{2P_1V_1}{\frac{33}{2}P_1V_1} = \frac{4}{33} = 12\%.$$

(b) The relative temperatures at various points around the cycle can be determined from the ideal gas law, $PV = NkT$. The lowest temperature occurs at the bottom-left corner when P and V are both smallest. As the pressure doubles during step A the temperature also doubles; then as the volume is tripled during step B so is the temperature. Thus the highest temperature, at the upper-right corner, is six times as great as the lowest temperature. For these extreme temperatures the maximum possible efficiency would be

$$e_{\max} = 1 - \frac{T_c}{T_h} = 1 - \frac{T_c}{6T_c} = \frac{5}{6} = 83\%.$$

The rectangular cycle is extremely inefficient compared to a Carnot cycle.

Problem 4.3. (Waste heat from a power plant.)

(a) An efficiency of 40% means that the other 60% of the energy consumed ends up as waste heat. That's 1.5 times as much as the amount that ends up as work. More generally, by the definition of efficiency and the first law,

$$e = \frac{W}{Q_h} = \frac{W}{Q_c + W},$$

so the waste heat is

$$Q_c = W\Big(\frac{1}{e} - 1\Big) = 1.5\,W = 1.5\text{ GW}.$$

(b) In one second, the waste heat dumped to the river is 1.5×10^9 J, and this heat is spread among 10^5 kg of water, so each kilogram gets 15 kJ. With a heat capacity of 4186 J/°C, the water's temperature increases by $\Delta T = Q/C = 15000$ J/4186 J/°C $= 3.6$°C.

(c) The latent heat to evaporate water is 2260 J/g (at 100°C). At room temperature it's about 8% more, as mentioned in Problem 1.54 and Figure 5.11; so I'll take $L =$ 2400 J/g. The total amount of water that must evaporate each second is then

$$\frac{1.5 \times 10^9 \text{ J}}{2400 \text{ J/g}} = 6 \times 10^5 \text{ g} = 600 \text{ kg}.$$

That's only 0.6 m^3, or only 0.6% of the water in the river.

Problem 4.4. (Engine driven by the ocean's thermal gradient.)

(a) Converting the temperatures to the kelvin scale, we get a maximum possible efficiency of

$$e = 1 - \frac{T_c}{T_h} = 1 - \frac{277 \text{ K}}{295 \text{ K}} = 0.061,$$

or about 6%.

(b) A rigorous calculation of the absolute minimum amount of water that we must process is not easy. As the engine extracts heat from the warm water, the water's temperature decreases and therefore so does the efficiency of the engine. To make a rough estimate, however, let's suppose that we extract heat from the warm water until its temperature drops by 9°C (half the temperature difference between the warm and cool water), and similarly that we expel heat into the cool water until its temperature increases by 9°C. Then the average temperatures of the reservoirs are 290.5 K and 281.5 K, so the efficiency is only

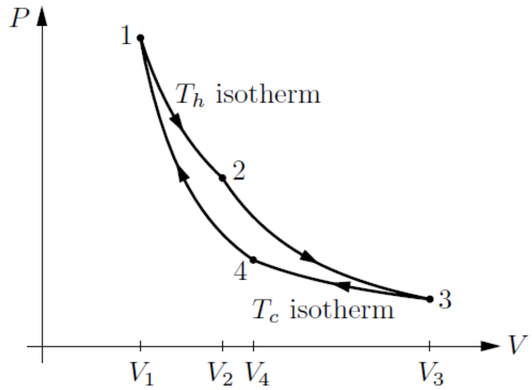
$$e = 1 - \frac{281.5}{290.5} = 0.031.$$

The heat extracted from each kilogram of the warm water is 9×4186 J $= 38$ kJ, but at 3.1% efficiency, this heat produces only 1.2 kJ of work. We need 10^9 J of work each second, so the amount of water required is

$$\frac{10^9 \text{ J}}{1200 \text{ J/kg}} = 8.6 \times 10^5 \text{ kg},$$

or about 900 cubic meters.

Problem 4.5. (Efficiency of an ideal gas Carnot engine.)



To compute Q_h and Q_c we need consider only the isothermal processes 1–2 and 3–4, since the other two steps are adiabatic. Furthermore, the heat input during an isothermal process is equal in magnitude to the work performed, since for an ideal gas $\Delta U \propto \Delta T = 0$. Therefore the heat input is

$$Q_h = |W_{12}| = \int_{V_1}^{V_2} P\,dV = NkT_h \ln\frac{V_2}{V_1},$$

and similarly,

$$Q_c = |W_{34}| = \int_{V_4}^{V_3} P\,dV = NkT_c \ln\frac{V_3}{V_4}.$$

The efficiency of the engine is

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h}\frac{\ln(V_3/V_4)}{\ln(V_2/V_1)},$$

which is equal to the Carnot efficiency provided that $V_3/V_4 = V_2/V_1$. To show that this is the case, note from equation 1.39 that for each of the adiabatic processes, $VT^{f/2}$ is constant

(where f is the number of degrees of freedom per molecule). For the adiabatic expansion 2–3, this implies

$$V_3T_c^{f/2} = V_2T_h^{f/2},$$

while for the adiabatic compression 4–1 we have

$$V_4T_c^{f/2} = V_1T_h^{f/2}.$$

Dividing these two equations, we obtain $V_3/V_4 = V_2/V_1$, as needed to cancel the logarithms in the preceding formula for the efficiency.

Problem 4.11. For the temperatures given, the maximum COP would be

$$\mathrm{COP} = \frac{T_c}{T_h - T_c} = \frac{0.01\ \mathrm{K}}{1\ \mathrm{K} - 0.01\ \mathrm{K}} = 0.01.$$

In other words, for each joule of heat extracted from the very cold reservoir, we must supply at least 100 J (or 99, to be precise) of work.

Problem 4.14. The heat pump is physically the same as an ordinary refrigerator, so please refer to the energy-flow diagram in Figure 4.4.

(a) The COP should be defined as the benefit divided by the cost. In this case the benefit is the heat that enters the building, Q_h, while the cost is the electrical energy consumed, W. So benefit/cost $= Q_h/W$.

(b) The energy in is $Q_c + W$ and the energy out is Q_h, so

$$Q_h = Q_c + W$$

under cyclic operation. The COP is therefore

$$\text{COP} = \frac{Q_h}{Q_h - Q_c} = \frac{1}{1 - Q_c/Q_h},$$

which is *always* greater than 1.

(c) The entropy expelled during the cycle must be at least as great as the entropy absorbed, so

$$\frac{Q_h}{T_h} \geq \frac{Q_c}{T_c} \quad \text{or} \quad \frac{T_c}{T_h} \geq \frac{Q_c}{Q_h}.$$

Because Q_c/Q_h must be *less* than or equal to T_c/T_h, the quantity $1 - Q_c/Q_h$ must be *greater* than or equal to $1 - T_c/T_h$, and therefore, by the result of part (b),

$$\text{COP} \leq \frac{1}{1 - T_c/T_h} = \frac{T_h}{T_h - T_c}.$$

(d) For an electric heater, all the electrical energy (W) is converted to heat (Q_h), so the COP is 1. An ideal heat pump, though, always has a COP greater than 1. For instance, if $T_h = 25°\text{C}$ and $T_c = 0°\text{C}$, then the COP can (in principle) be as high as $298/25 \approx 12$. In practice the COP is never this high, but as long as T_h and T_c aren't too different, a heat pump offers a big advantage in efficiency over an electric heater. On the other hand, a heat pump is more expensive to manufacture and maintain, since it a complicated device with many moving parts. Fortunately, a central air conditioning system can double as a heat pump in the winter. So if you're already planning to install central air, and your winters aren't *too* cold, get a heat pump.