PHY 3107, Spring 2018, Homework #3

due Thursday, Feb. 1.

This homework is likely to be the most mathematically challenging for the term – don't worry if it seems tough! But get help from the SPS room, peers and of course **me**(!).

1.) In class, the slide showed the z-component of the angular momentum was given by $L_z = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$ (Eqn. 8.20 in the text), or in spherical coordinates, the operator is given as $[L_z] = -i\hbar\frac{\partial}{\partial\varphi}$ (Eqn. 8.24), using $z = r\cos\theta\,, x = r\sin\theta\cos\varphi\,, y = r\sin\theta\sin\varphi$. Now show that the spherical representation for the other two components is given by $[L_x] = i\hbar\left(\sin\varphi\frac{\partial}{\partial\theta} + \cot\theta\cos\varphi\frac{\partial}{\partial\varphi}\right)$ and $[L_y] = -i\hbar\left(\cos\varphi\frac{\partial}{\partial\theta} - \cot\theta\sin\varphi\frac{\partial}{\partial\varphi}\right)$ as given by Eqn. 8.23. [Hint see also pages 274 and 275 of your text. This problem is meant to explore both operators and going back and forth between Cartesian and spherical coordinate systems.]

2.) Calculate the probability that the electron is found inside a distance of half the Bohr radius of a_0 for the ground state of hydrogen (e.g. R_{10}). Remember to use $d^3x = r^2\sin\theta\ dr\ d\theta\ d\varphi$ in your integral, and set the limits on r from 0 to $a_0/2$.

3.) A hydrogen atom has an electron in the wavefunction $\psi_{nlm} = R_{32}\left(\sqrt{\frac{1}{2}}Y_2^1 + \sqrt{\frac{1}{6}}Y_2^0 + \sqrt{\frac{1}{3}}Y_2^{-1}\right)$. A) What is the electron's energy? B) What are the possible values if L^2 is measured? C) If we also measure L_Z, what would be the possible values and what would the probability be for each one? What would the expectation value be of L_Z? [Note: the only potential is due to the static Coulomb field of the nucleus. This should be an easy problem.]

4.) Show that the function $R(r) = Ae^{-br}$ satisfies the radial equation for hydrogen (equation 8.17, with $U(r) = -kZe^2/r$). Use your solution to find the angular momentum and energy for this wavefunction. [Hint: This function will be an eigenfunction for only certain values of b.]

5.) Ignoring spin, an electron is known to be in a hydrogen atom state given by the wavefunction $\psi(r,\theta,\varphi) = \sqrt{\frac{2}{3}}R_{21}Y_1^0 + cR_{32}Y_2^{-1}$. a) Pick a value of c which normalizes the wavefunction. b) What possible outcomes (with what associated probabilities) are there for a measurement of the energy? c) What possible outcomes (with what associated probabilities) are there for a measurement of the operator $\hat{L}^2$? d) Suppose you measure $\hat{L}_z$ and find a value of $-\hbar$. Write a properly normalized wavefunction for the particle immediately after this measurement. e) Now let's add spin. For the wavefunction you found in part (d), what are the possible values of the total angular momentum of the electron, $\vec{J} = \vec{L} + \vec{S}$?