PHYS 4134, Fall 2016, Homework #1

Due Friday, August 26, at the start of class.

This first set is LONG! Your future homework sets will NOT be this long, so don't get too worried about the time this class will take you.

1. Equation 1.31 suggests that the experimentally determined radius of a nucleus is $R = 1.41\ A^{1/3} + 2.11\ fm$ as found from Rutherford scattering. A) Assuming that a nucleus is a sphere of nuclear matter of radius R, find the average nuclear density in SI units. [Hint: for part A ignore the constant term of 2.11 fm and take $R = 1.41\ A^{1/3}\ fm$.] B) What does the factor of 2.11 fm represent in equation 1.31? [Hint: it has to do with the fact that this radius comes from Rutherford scattering.] C) Why does the radius go like $A^{1/3}$?

2. A typical nuclear length scale is a Fermi, or femtometer (fm). A typical atomic length scale is an Angstrom, and a molecular length scale can be nanometers. Find the wavelength for γ rays, electrons, and alpha particles of energy: a) 4 MeV, b) 5 eV, and c) 0.025 eV.

3. The Heinsenberg Uncertainty principle tells us that if we measure the location of a particle with precision Δx and simultaneously measure the momentum with precision Δp_x then the product of the two uncertainties can never be smaller than $\hbar/2$: $\boldsymbol{\Delta x\ \Delta p_x \geq \hbar/2}$. There is a LOT of philosophical implications to this statement, but let's stick with the physics of it. A) A neturon confined inside a cubic box has a kinetic energy of 12 MeV. What is the minimum size of the box?

The next part of the set looks at cross sections, counts & counting rates, and luminosity. For those of you who took Modern Physics II in the Spring of 2016, this is a repeat of the material – so you had better know it! This homework set uses algebra – <u>there are no integrals needed</u>. Because I work mostly at accelerators, i
uses examples from accelerators but the concepts are universal. The idea occurs in astronomy, cosmology, nanotechnology, atomic beam collisions, and other experiments.

In the 1940s, the phrase "as big as a barn" was used to define a new unit of area, 1 barn = 10^{-28} m^2. Although a barn seems like a really small size, we often measure smaller things, making the unit of a barn huge by comparison.

If the "yield" or number of counts observed in an experiment is N, and the luminosity $\mathcal{L}$ is the product of the how thick the target is multiplied by the number of incident particles in the beam, then we can write:

$$N = d\sigma/d\Omega * \mathcal{L} * Acceptance$$

I.e., we see more counts if we have a bigger cross section, a larger luminosity, or a bigger solid angle/acceptance. The physics is hidden in the differential cross section, $d\sigma/d\Omega$, but the measurements involve the mundane aspects of the luminosity and acceptance too.

The luminosity for an experiment where a beam of particles strikes a target (as opposed to having two beams strike each other) can be written as:

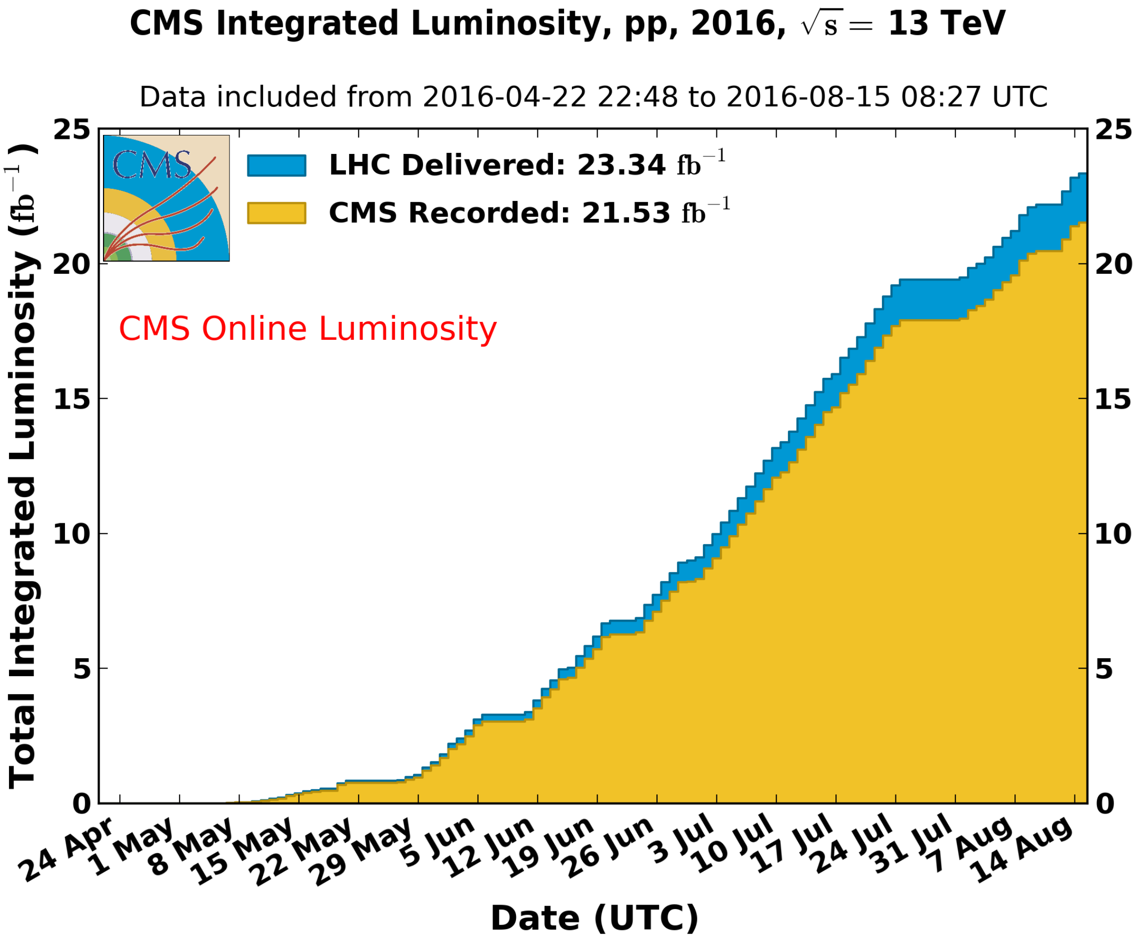
$$\mathcal{L} = target * beam = \left(\rho\ * t * \frac{N_A}{M}\right) * \left(I * \frac{1\ C/s}{A} * \frac{1}{e}\right)$$

where the target is found from ρ (the target density in $\frac{g}{cm^3}$), t is the target thickness in cm, M is the Molar mass in $\frac{g}{mole}$, and N_A is Avagadro's number $\frac{particles}{mole}$. The beam is given in terms of the current in Amperes, and the charge of the beam particles (in this case, I assumed the particles to be electrons or protons of charge e).

4. Jefferson Lab's Halls A and C are where I do a lot of research, along with Drs. Boeglin and Reinhold. One of the experiments there used a deuterium target that was 5.0 cm long, and beam currents upto 120 microAmperes (or μA). What was the luminosity, L, at 120 μA? [The density of the liquid deuterium is 0.167 g/cm^3.]

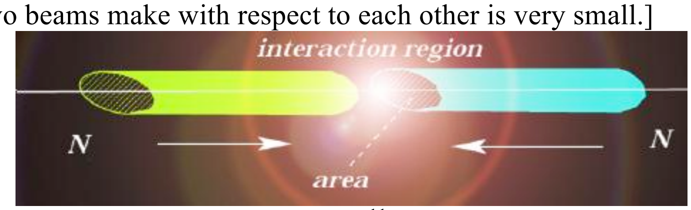
5. Drs. Raue and Guo often do experiments in Jefferson Lab's Hall B facility. The detector (shown above) is called CLAS (for the CEBAF Large Acceptance Spectrometer. CEBAF is an acronym for the Continuous Electron Beam Accelerator Facility, the name of the accelerator at JLab). CEBAF accelerates bunches of electrons in sharp bursts that are only about 2 picoseconds long. However between those bunches, there are 667 picoseconds, or 0.667 ns. The beams get sent to 3 halls, meaning each hall gets a burst every 2 ns. In Hall B, average currents as low as 200 picoAmperes are used.
   a. If the beam comes in pulses every 2 ns, how many electrons are in 1 bunch?

b. How many electrons come every second?



The figure above represents how many proton collisions were delivered in 2016 by CERN's Large Hadron Collider (LHC) to the CMS experiment. [Note that sometimes our experiment was not ready to take data, so the CMS recorded luminosity is less than the LHC accelerator delivered luminosity.] What is plotted is the cumulative luminosity, L, (on the y-axis) vs. the calendar day delivered to CMS (blue) and recorded by CMS (orange) during stable beams and for p-p collisions at 13 TeV centre-of-mass energy in 2016. The y-axis is labeled "Total Integrated Luminosity (fb^{-1})". That means the luminosity is in units of inverse femto-barns, and has been summed from the first day of running the accelerator, 1=24 April 2016 until the final day for the summer, August 15, 2016. [Femto is the abbreviation for 10^{-15}, making 1 fb = 10^{-43} m^2 or equivalently 1 fb = 10^{-39} cm^2 .

Let's look at how the CMS luminosity is calculated. There is no longer a fixed target. Instead, the beams scatter from each other. Each beam is made of bunches of protons, which circulate in opposite directions in the accelerator. [The two beams are offset during their orbits, except at each of the 4 points where one of the experiments is located. Obviously the two beams cannot quite collide head-on, but the angle the two beams make with respect to each other is very small.]



Each bunch of protons contained as many as 1.15×10^{11} protons, and each proton might collide with any proton from the other bunch. The bunches come quickly – every 25 nanoseconds(!). Finally, each beam has a size, and looks to the other bunch like a (Lorentz-boosted) flattened disk of radius 16 microns. [Remember, a typical human hair is 50-100 microns, so this is a skinny beam.] Then if N_1 and N_2 are the number of protons in a bunch for beams 1 and 2, t is the time between bunch crossings, and S_{eff} is the effective area of each beam:

$$\mathcal{L} = N_1 N_2 / (\tau \times S_{eff}) = f\, N_1 N_2 \,/ (4\pi r^2)$$

Now $f = 1/\tau = 40 \times 10^6$ is the frequency, and the additional factor of 4 in the area of the beam comes from using defining the beam radius as being one standard deviation from the centroid of the beam. [Like earth's atmosphere or the atom's electron cloud, the "size" of the beam is not perfectly defined.] This factor of 4 standard deviations is a kind of safety factor – 99.9936656% of the beam will be within 4 standard deviations if the beam distribution is a Gaussian.

a) Put the numbers in, assuming $N_1 = N_2 = 1.15\times10^{11}$. Give the value for the luminosity in cm^{-2} s-^{1}:

b) Of course the chart above is integrating over time. To get the final, integrated luminosity for CMS of 21.53 fb^{-1}, you multiply the time. Let's do it the other way though – tell me how many days CMS ran at the equivalent of full luminosity (e.g, take your answer from part a which is a luminosity per second and convert to $fb^{-1}day^{-1}$ to get the "luminosity per day". Then divide the integrated luminosity by the luminosity per day).

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