PHYS 4134, Fall 2015, Homework #2

Due Wednesday, September 9, at 4:00 pm

1. Rutherford scattering sends an incident alpha particle scattering from a nucleus. In class, we did not have time to think about the difference between observing the reaction in the laboratory frame where the target is at rest, and the reaction observed in the center-of-mass frame (CM) where the total initial (and final) momentum are zero.

   In general, given 2 particles with masses $m_1$and m_2 (with m_2 initially at rest) with coordinates $\vec{r}_1$ and $\vec{r}_2$ the equations of motion can be written as $m_1\ddot{\vec{r}}_1 = -\nabla_1 U|\vec{r}_1 - \vec{r}_2|$, and $m_2\ddot{\vec{r}}_2 = -\nabla_2 U|\vec{r}_1 - \vec{r}_2|$. We will use θ_{Lab} as the outgoing angle of particle 1 in the laboratory frame measured with respect to the incident direction.

   In the CM it is more common to use the equivalent two variables $\vec{r} = \vec{r}_1 - \vec{r}_2$and $\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$ instead. This gives a velocity of the center of mass frame (as measured in the laboratory frame) of $\vec{v}_{CM} = \dot{\vec{R}}_{CM} = \frac{m_1\vec{v}_1}{m_1+m_2}$ (remember $\vec{v}_2$= 0 initially, and conservation of momentum means the center of mass velocity is a constant of the motion). And that means for non-relativistic velocities, $\vec{v}_{1,CM} = \vec{v}_1 - \vec{v}_{CM} = \frac{m_2 v_1}{m_1+m_2}$ and for the second particle $\vec{v}_{2,CM} = \vec{v}_2 + \vec{v}_{CM} = 0 + \vec{v}_{CM} = \frac{m_1 v_1}{m_1+m_2}$.

   Assuming elastic scattering, we can say the magnitudes of the velocities are unchanged after collision but the outgoing angles of particles 1 and 2 will depend on the dynamics. I.e., looking at particle 1 after the collision, along the initial direction we have $v_1 \cos\theta_{Lab} - v_{1,CM} = v_{1,CM}\cos\theta_{CM}$ or $v_1\cos\theta_{Lab} = v_{1,CM} + v_{1,CM}\cos\theta_{CM}$ and $v_1\sin\theta_{Lab} = v_{1,CM}\sin\theta_{CM}$ for the perpendicular component.

   Dividing the two gives $\tan\theta_{Lab} = \frac{\sin\theta_{CM}}{\cos\theta_{CM} + (\frac{v_{CM}}{v_{1,CM}})} = \frac{\sin\theta_{CM}}{\cos\theta_{CM}+\zeta}$ where this defines the $\zeta \equiv \frac{v_{CM}}{v_{1,CM}}$. It even turns out to be able to re-write this (here I am skipping the steps of inverting the equation and substituting for v) as $\cos\theta_{Lab} = \frac{\cos\theta_{CM}+\zeta}{(1+2\zeta\cos\theta_{CM}+\zeta^2)^{1/2}}$.

   Now for the homework! This is meant to be your first use of Mathematica, so I am taking "baby steps". Use Mathematica (or another program) to sketch the Cosine of the laboratory angle as a function of the Cosine of the center of mass angle, for the nonrelativistic elastic scattering of particles of unequal mass, for the two cases: a) when $\zeta = 0.05$ and b) when $\zeta = 0.05$. Note this corresponds to ratios of 1:20 and 20:1 for the masses of particles 1 and 2.

   To get you started, see the modify the Mathematica notebook at http://faculty.fiu.edu/~markowit/WidelyApplied/Homework/HW2-1.nb .

2. A thin (1.00±0.01 mg/cm^2) target of ^{48}Ca is bombarded with a (10±0.15)-nA beam of α particles. A detector, subtending a solid angle of (2.00±0.02)×10^{-3} steradians, records 15 protons per second.
   a) If the angular distribution is measured to be isotropic, determine the total cross section (in mb) for the ^{48}Ca(α,p) reaction. Take the atomic mass of ^{48}Ca to be 48 u.
   [Hint: The measured yield (or "rate" in your text) is given by $R = \frac{d\sigma}{d\Omega}\,\Delta\Omega\,\frac{(\rho t)N_A}{M_A^{mol}}\,I$.]

   b) How long would one have to measure to obtain the cross section with a relative error of 3%. Assume that the error in the measurement of time is negligible. (For a 3% statistical uncertainty, it would need $\frac{1}{\sqrt{N}} = 0.03$ where N is the number of counts. However here you need to add in quadrature the uncertainties from the target thickness, beam current, and solid angle.)

3. Derive the formula $N(t) = P(1 - e^{-\lambda t})/\lambda$ for the production of a radioactive nucleide (with decay constant λ) as a function of time, given that the production rate is constant at P nuclei per second.

Estimate the time it will take to produce a 100 nCi source of ^{14}C by irradiating 1 g of natural carbon in a neutron flux of 10^{14} cm^{-2}s^{-1}. The natural abundance of the two stable carbon isotopes ^{12}C and ^{13}C is 98.90% and 1.10% respectively. The cross section for the neutron capture reactions ^{12}C(n,γ)^{13}C and ^{13}C(n,γ)^{14}C are 3.4 mb and 0.9 mb, respectively.

4. [Hint: Before attempting this problem go over *Appendix E: Coulomb Scattering* of your book.] Fig. 1 shows the cross section for elastic scattering of alpha particles by tantalum at 60° as a function of alpha particle energy. At an energy of 23.9 MeV, the observed cross section starts to deviate from the expected behaviour for Coulomb scattering. This would be expected, if the alpha particle comes close enough to the tantalum nucleus that the process gets effected by the strong nuclear force. At 60° and 23.9 MeV, what is the distance of closest approach (apsidal distance), d, of the classical alpha trajectory.

- What is the Coulomb energy (potential) of an alpha particle and a tanta- lum nucleus at the distance d. (Please don't use numbers yet. E.g., just refer to the charge of the particles as Z_1e and Z_2e.)

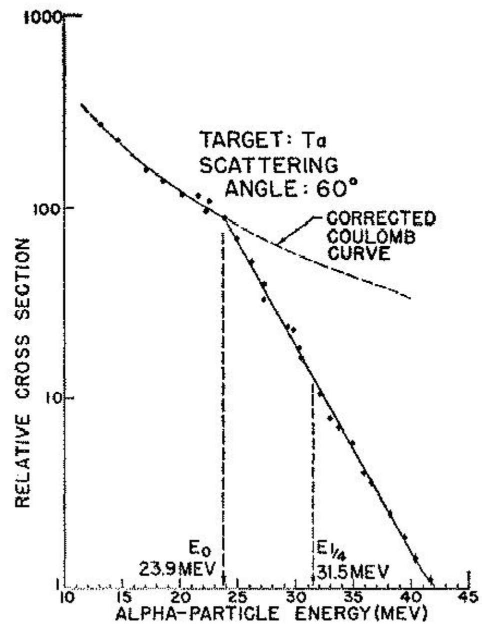
- With the simplification that the tantalum nucleus stays at rest (is infinitely heavy compared to the alpha particle), what is the energy of the alpha particle , E', at distance d, if the beam energy was E.

- Let $\vec{p}'$ be the momentum vector of the alpha particle a closest approach (distance d). What is the alpha particles angular momentum at this point.

- Find the ratio p'/p and relate it to E'/E.

- Find d as a function of the known parameters E, Z_1, Z_2, and the scattering angle Θ. Simplify as much as possible.

- Calculate the distance of closest approach in fm. (This is the first time you should need a claculator!)



- Compare your result with Eq. 1.31 in your book.

Figure 1: Elastic scattering of alpha particles by tantalum at 60° [1].

References

[1] G.W. Farwell and H.E. Wegner, Phys. Rev. 95, 1212 (1954).