

PHZ 3113 – PROBLEM SET 1

- 1) A particle moves so that its position vector is given by the equation

$$\mathbf{r}(t) = r \cos(\omega t) \mathbf{x}\text{-hat} + r \sin(\omega t) \mathbf{y}\text{-hat}$$

where $\mathbf{x}\text{-hat}$ and $\mathbf{y}\text{-hat}$ are the unit vectors along the x and y-axes, and ω is a constant.

- a) Evaluate the vector $d\mathbf{r}/dt$ assuming that the radial distance r depends on time and then use the result to show that $\mathbf{r} \times (d\mathbf{r}/dt) = \omega r^2 \mathbf{z}\text{-hat}$
- b) Show that if r does *not* depend on time, $d^2\mathbf{r}/dt^2 = -\omega^2 \mathbf{r}$.

- 2) **Text Problem 2.9**

- 3) **Text Problem 2.12**

- 4) The force exerted by a linear oscillator in two dimensions is given by the equation

$$\mathbf{F} = -kx \mathbf{x}\text{-hat} - ky \mathbf{y}\text{-hat}$$

where $\mathbf{x}\text{-hat}$ and $\mathbf{y}\text{-hat}$ are the unit vectors along the x and y-axes, and k is a constant. The work done *against* this force is given by the line integral

$$W_{\text{against}} = -\int \mathbf{F} \cdot d\mathbf{r}$$

evaluated from the initial position to the final position. Calculate W_{against} from the position $x=1, y=1$ to position $x=4, y=4$ along each of the following paths:

- a) $x=1, y=1 \rightarrow x=4, y=1 \rightarrow x=4, y=4$
- b) $x=1, y=1 \rightarrow x=1, y=4 \rightarrow x=4, y=4$
- c) $x=1, y=1 \rightarrow x=4, y=4$ along the line $y=x$

5) **Text Problem 3.2** – note that a field is conservative if it has zero curl. In the second part, you need to find a scalar function $\phi(x,y,z)$ whose x, y, and z partial derivatives are equal to the x, y, and z components of the field. Note that in integrating a function of three variables over one of the variables, the constant of integration is actually a *function* of the other two variables. Finally, note in the last part, that $\nabla\phi\cdot\mathbf{r}=d\phi$, a perfect differential.

6) **Text Problem 3.3**

7) Making use of Stoke's theorem, show that for any two scalar functions u and v,

$$\oint (u\nabla v + v\nabla u) \cdot d\mathbf{r} = 0$$

where the integral is around any closed curve.

8) **Text Problem 3.28** – note that the specified loop is traversed in the *counterclockwise* direction, so that the normal to the plane of the loop is in the *positive z*-direction. Thus, to use Stokes' Theorem, you only need to evaluate the z-component of the curl. Note that the evaluation of the final integral over the y-coordinate requires an integration by parts.