Spring 2022 – MAP4401
Homework I

This homework is due by **Tuesday February 15**. (I will not accept late returns). Clarity of exposition is expected and no credits will be given for sloppy work. I trust that your work will be individual and you are free to consult notes and books. You have to turn in the following 9 problems.

Problem 1. Write in formal style the BVP for the temperature in a sphere of radius R made up of material with diffusivity K and such that the upper hemisphere is kept at constant temperature T_1 and the lower hemisphere is kept at a constant temperature T_2. Use spherical coordinates in the sphere.

Problem 2. Write in formal style the BVP for the temperature in a circular plate (disk) of radius R made up of material with diffusivity K and such that one half of the boundary (say the upper semicircle) is kept at constant temperature T_0, and the other half of the boundary (lower semicircle)is insulated. Assume that the disk is laterally insulated (so there is no heat flow in the direction perpendicular to the disk). Use polar coordinates in the disk.

Problem 3. Write in formal style the BVP for the vibrations of a string subject to the following. The string has length L (in centimeters); its end points are held fixed; there is a damping with coefficient a (resistance to the motion); assume that initially the string is pulled by 1 cm at the point $x = L/4$ and then released from rest; (for simplicity assume that the speed of the wave c is normalized to be 1)

Problem 4. Write in formal style the BVP for the vibrations of a drum head (membrane) subject to the following. The drum head is a circular membrane of radius R with a fixed boundary. Assume that the motion of the drum (which was at rest) is started by hitting it at its center with a circular object of radius $R/10$ which has a downward velocity of 2cm/sec. (Assume the speed of the wave is normalized to be 1).

Problem 5. Use the method of separation of variables to solve the BVP

$$\begin{aligned} &u_t(x,t) = 2u_{xx}(x,t) && 0 < x < 5, \quad t > 0 \\ &u_x(0,t) = u_x(5,t) = 0 && t > 0 \\ &u(x,0) = 100 - 7\cos\frac{3\pi x}{5} && 0 < x < 5 \end{aligned}$$

Problem 6. Use the method of separation of variables to solve the BVP

$$\begin{aligned} &u_{tt}(x,t) + (0.2)u_t(x,t) + (0.01)u = (0.01)u_{xx}(x,t) && 0 < x < \pi, \quad t > 0 \\ &u(0,t) = u(\pi,t) = 0 && t > 0 \\ &u(x,0) = 0 && 0 < x < \pi \\ &u_t(x,0) = \sin 3x - \frac{1}{5}\sin 7x && 0 < x < \pi \end{aligned}$$

Problem 7. Use the method of separation of variables to solve the BVP

$$\begin{aligned} &\Delta u(x,y) = 0 && 0 < x < \pi, \quad 0 < y < \pi \\ &u_x(0,y) = u_x(\pi,y) = 0 && 0 < y < \pi \\ &u(x,0) = 0 && 0 < x < \pi \\ &u(x,\pi) = \cos 3x - \frac{1}{5}\cos 9x && 0 < x < \pi \end{aligned}$$

Problem 8. Use the method of separation of variables to solve the BVP given in polar coordinates by

$$\begin{aligned} &\Delta u(r,\theta) = 0 && 0 < r < 2, \quad 0 \le \theta \le 2\pi \\ &u_r(2,\theta) = \sin\theta + \cos(5\theta) && 0 \le \theta \le 2\pi \end{aligned}$$

Problem 9. Use the method of separation of variables to solve the BVP

$$\begin{aligned} &u_t(x,y,t) = \Delta u(x,y,t) && 0 < x < \pi, \quad 0 < y < 2\pi, \quad t > 0 \\ &u(0,y,t) = u(\pi,y,t) = 0 && 0 < y < 2\pi, \quad t > 0 \\ &u_y(x,0,t) = u_y(x,2\pi,t) = 0 && 0 < x < \pi, \quad t > 0 \\ &u(x,y,0) = \sin x + \cos\frac{y}{2}\sin 2x && 0 < x < \pi, \quad 0 < y < 2\pi \end{aligned}$$