III. Obstructions to Solvability

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Problem. Given $\lambda : \partial \mathbb{D} \longrightarrow \partial \mathbb{D}$, Hölder continuous, find $w \in C(\overline{\mathbb{D}})$ holomorphic in \mathbb{D} such that $\operatorname{Re}(\overline{\lambda} w) = 0$ on $\partial \mathbb{D}$.

The problem has nontrivial solutions only when the index κ of λ is \geq 0.

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$$W(Z) = Z^{\kappa} e^{i\gamma(Z)} \left(id_0 + \sum_{j=1}^{\kappa} d_j Z^j - \overline{d_j} \frac{1}{Z^j} \right)$$

with $d_0 \in \mathbb{R}$, $d_j \in \mathbb{C}$, and $\gamma(z) = S(\arg(z^{-\kappa}\lambda(z)))$

$$S(\psi)(z) = \frac{1}{2\pi i} \int_{\partial \mathbb{D}} \psi(\zeta) \frac{\zeta + z}{\zeta - z} \frac{d\zeta}{\zeta}$$

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$L = A \frac{\partial}{\partial x} + B \frac{\partial}{\partial y}$, locally solvable, nonsingular, C^{ω} , in $\widetilde{\Omega}$, open subset of \mathbb{R}^2 .

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Allow *L* to have one dimensional orbits. $\Omega \subset \subset \widetilde{\Omega}, \Omega$ simply connected. $\Lambda : \partial\Omega \longrightarrow \partial\mathbb{D},$ Hölder continuous. **Problem 1.** Lu = 0 in Ω , and Re $(\overline{\Lambda} u) = 0$ on $\partial\Omega$. **Problem 2.** Lu = 0 in Ω , and Re $(\overline{\Lambda} u) = \phi$ on $\partial\Omega$.with $\phi \in C^{\alpha}(\partial\Omega, \mathbb{R}).$ **Problem 3.** Lu = f in Ω , and Re $(\overline{\Lambda} u) = 0$ on $\partial\Omega$.with $\phi \in C^{\alpha}(\partial\Omega, \mathbb{R})$ and $f \in C^{c}(\overline{\Omega}).$

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$$\Omega \setminus \bigcup_{j=1}^{N} \Gamma_{j} = \Omega_{1} \cup \cdots \cup \Omega_{N+1}$$

$$\partial \Omega_{j} = A_{j1} \cup \Gamma_{j1} \cup A_{j2} \cup \Gamma_{j2} \cup \cdots \cup A_{jm_{j}} \cup \Gamma_{jm_{j}}$$

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Associate indices to $\Lambda \in C^{\alpha}(\partial\Omega, \partial\mathbb{D})$

To each connected component Ω_j , associate and $\kappa_j \in \mathbb{Z}$. Jump of Λ along orbit $\Gamma_{jk} = \operatorname{arc}(p_{jk}^-, p_{jk}^+)$: $\vartheta_{jk} = \arg \Lambda(P_{jk}^-) - \arg \Lambda(P_{jk}^+)$.

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 $n_j = \#$ of orbits on $\partial \Omega_j$ with q_{jk} odd. Define

$$\kappa_j = \operatorname{Ind}\left(\Lambda, \partial \Omega_j\right) = \frac{1}{2} \left(\sum_{k=1}^{m_j} q_{jk} - n_j\right)$$

 $\delta_j = \#$ of orbits with $\alpha_{jk} = 0$.

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Theorem (A. Ainouz, K. Boutarene, A.M (2014))

Problem Lu = 0 in Ω , $Re(\overline{\Lambda} u) = 0$ on $\partial\Omega$ has nontrivial solutions if and only if there is j with $2\kappa_j \ge \delta_j$. Furthermore, the number of independent solutions is

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$$r = \sum_{2\kappa_j \ge \delta_j} (2\kappa_j - \delta_j + 1)$$

Theorem (A. Ainouz, K. Boutarene, A.M (2014))

Assume $\delta_j = 0$ for all *j*. Problem Lu = 0 in Ω , $Re(\overline{\Lambda} u) = \phi$ on $\partial\Omega$ has solution $u \in C^{\mu}(\overline{\Omega})$ except (possibly) at isolated points. The singular points of *u* consist of a single pole of order $-\kappa_j$ in each component Ω_j with $\kappa_j < 0$. Moreover, the value *u* on the orbit $\Gamma = arc(p^-, p^+)$ is uniquely determined:

$$u(\Gamma) = (-1)^q \frac{\Lambda(p^-)\phi(p^+) - \Lambda(p^+)\phi(p^-)}{i\sin(\pi\alpha)}$$

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Assume $\delta_j = 0$ for all *j*. Problem Lu = 0 in Ω , $Re(\overline{\Lambda} u) = \phi$ on $\partial\Omega$ has solution $u \in C^{\mu}(\overline{\Omega})$ except (possibly) at isolated points. The singular points of *u* consist of a single pole of order $-\kappa_j$ in each component Ω_j with $\kappa_j < 0$. Moreover, the value *u* on the orbit $\Gamma = arc(p^-, p^+)$ is uniquely determined:

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$$L = (1 + 2ixy)\frac{\partial}{\partial y} - i(x^2 - 1)\frac{\partial}{\partial x}.$$

x = 1 and x = -1 are one dimensional orbits of *L*. First integral $Z(x, y) = x + iy(x^2 - 1)$. Let $\Omega = D(0, 2)$, $\Omega \setminus \{x = \pm 1\} = \Omega_1 \cup \Omega_2 \cup \Omega_3$ with $\Omega_1 = \Omega \cap \{x > 1\}, \Omega_2 = \Omega \cap \{-1 < x < 1\}, \Omega_3 = \Omega \cap \{x < -1\}$. Let $\Lambda(2e^{i\theta}) = e^{if(\theta)}$, with

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Indices

$$\kappa_1 = \frac{1}{2}(q_0 + q_1 - n_1), \quad \kappa_2 = \frac{1}{2}(q_1 + q_2 - n_2), \quad \kappa_3 = \frac{1}{2}(q_2 - n_3)$$

• Case $\Lambda(2e^{i\theta}) = e^{i\theta}$:

 $\kappa_1 = 0, \ \kappa_2 = -1, \ \kappa_3 = 0, \ \alpha_1 = \alpha_2 = 2/3, \ \delta_1 = \delta_2 = \delta_3 = 0$ Problem Lu = 0 in D(0, 2), Re $(e^{-i\theta}u)$ = 0 has 2 independent solutions. Both are constant in Ω_2 .

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Problem Lu = 0 in D(0, 2), Re $(e^{-r \sin \theta} u)) = 0$ has 1 independent solutions constant in $\Omega_2 \cup \Omega_3$.

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<u>§</u>3

 \mathcal{E}^{n+1} : space of germs at $0 \in \mathbb{R}^{n+1}$ of \mathbb{C} -valued C^{∞} functions. $\omega \in \Lambda^1(\mathcal{E}^{n+1})$ with $\omega(0) \neq 0$ and $\omega \wedge d\omega = 0$.

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$$\begin{split} \omega &= dx + \omega' = dx + \sum_{j=1}^{n} \alpha_j(x, t) dt_j. \\ L_j &= \frac{\partial}{\partial t_j} - \alpha_j \frac{\partial}{\partial x}, \quad j = 1, \cdots, n. \\ \Lambda^p_{\omega}(X) &\cong \Lambda^{p,0}(X) = \{ f \in \Lambda^p(X) : f = \sum_{|J| = p} f_J(x, t) dt_J \}. \\ \mathbb{L} : \Lambda^{p,0}(X) \longrightarrow \Lambda^{p+1,0}(X), \quad \mathbb{L} = d_t - \omega' \wedge \frac{\partial}{\partial x} \\ \mathbb{L}f &= \sum_{|J| = p} \left(d_t f_J - \frac{\partial f_J}{\partial x} \omega' \right) \wedge dt_J = \sum_{|M| = p+1} \left(\sum_{i \in M} (-1)^{\sigma(i,M_j)} L_j f_{M_j} \right) dt_M \end{split}$$

from now on we assume ω exact

$$\omega = d(x + i\phi(x, t))$$

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$$\omega = dx + \omega' = dx + \sum_{j=1}^{n} \alpha_j(x, t) dt_j.$$

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$$\Lambda^p_{\omega}(X) \cong \Lambda^{p,0}(X) = \{f \in \Lambda^p(X) : f = \sum_{|J| = p} f_J(x, t) dt_J\}.$$

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<u>§</u>3

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with $\phi \in \mathcal{E}^{n+1}$ and \mathbb{R} -valued.

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Equation $\mathbb{L}u = f$ can be solved with $u \in \Lambda^{p-1,0}(\mathcal{E}^{n+1})$ for every $f \in \Lambda^{p,0}(\mathcal{E}^{n+1})$ satisfying $\mathbb{L}f = 0$

84

He conjectured that the triviality of $H^{\rho}_{\omega}(\mathcal{E}^{n+1})$ is equivalent the triviality of (p-1) homology group of the generic fiber of $x + i\phi(x, t)$.

The completed proof of the conjecture was given by Cordaro and Hounie in 2001.

Some articles that were written about the problem include Cordaro-Treves (1991): " \implies " for all *p* Mendoza-Treves (1991): " \Leftarrow " for p = 1Chanillo-Treves (1997): " \Leftarrow " for all *p* when ω is real analy Cordaro-Hounie (1999): " \Leftarrow " for p = n

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Ideas in [A.M] 1997 can used for $\omega = d(x + i\phi(t))$ where $\phi \in \mathcal{E}^n$ has an algebraically isolated singularity at $0 \in \mathbb{R}^n$. Let $L = |\phi|^{-1}(c)$ with c > 0 small. Then

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 $H^p_\omega(\mathcal{E}^{n+1})\cong H^{p-1}(L)\otimes \mathcal{S}$

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Ideas in the proof

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$\mathrm{H}^{p}_{\omega}(\mathcal{E}^{n+1}) \cong \mathrm{H}^{p}_{\omega}(\mathcal{F}_{0}(\mathbb{R} \times V))$

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Reduction to wedge

$$\mathrm{H}^{p}_{\omega}(\mathcal{F}_{0}(\mathbb{R}\times V))\cong \oplus_{i=1}^{N}\mathrm{H}^{p}_{\omega}(\mathcal{C}_{0}(W_{i}))$$

where W_1, \dots, W_N are components of $\mathbb{R}^{n+1} \setminus \mathbb{R} \times V$ and $C_0(W_i)$ is the space of germs at 0 of $C^{\infty}(\overline{W_i})$ that are flat along $\overline{W_i} \cap \mathbb{R} \times V$.

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Two germs f_1, f_2 of C^{∞} at $0 \in \overline{\mathbb{R}^2_+}$ $(y \ge 0)$ are equivalent if $\int_{\mathbb{C}} \frac{f_1(\zeta) - f_2(\zeta)}{\zeta - x} d\zeta \wedge d\overline{\zeta} \quad \text{is real analytic function } (x \in \mathbb{R})$

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 ${\mathcal S}$: space of equivalence classes, ${\mathcal S}={\mathcal C}^\infty({\mathbb R}^2_+,0)/{\sim}.$

Lemma

$$H^1_{d(x+iy)}(\mathcal{F}_0(\{y=0\}))\cong S$$

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$$u_0(z) = \frac{-1}{2\pi i} \int_{\mathbb{C}} \frac{f(\zeta)}{\zeta - z} \, d\zeta \wedge d\overline{\zeta}$$

then in order for *u* to be flat along y = 0 we need $u_0(x) = h(x)$.

$$\Phi: \operatorname{H}^{1}_{d(x+iy)}(\mathcal{F}_{0}(\{y=0\})) \longrightarrow \mathcal{S}, \quad \Phi([\eta]) = [f]$$

To $\eta = fd\overline{z} + gdz \in \Lambda^1(\mathcal{C}^{\infty}(\overline{\mathbb{R}^2_+}) \text{ associate } [f] \in \mathcal{S}.$

Since the general solution of $u_{\overline{z}} = f$ in $y \ge 0$ is $u(z) = u_0(z) - h(z)$, with *h* holomorphic in y > 0 and smooth up to $y \ge 0$ and

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$F(x,t,s) = x + i(\phi(t) + s).$

$$\begin{split} &\mathcal{E}_{x} : \text{germs at } 0 \in \mathbb{R} \text{ of } C^{\infty} \text{ functions} \\ &\mathcal{E}_{x}[t,s] : \text{ring of formal power series in } t,s \text{ with coefficients in } \mathcal{E}_{x}. \\ &\Lambda^{\bullet}_{dF}(\mathcal{E}_{x}[t,s]) = \frac{\Lambda^{\bullet}(\mathcal{E}_{x}[t,s])}{\Lambda^{\bullet-1}(\mathcal{E}_{x}[t,s]) \wedge dF} \end{split}$$

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Proposition

 $H^0_{dF}(\mathcal{E}_x[t,s]) \cong \mathcal{E}_x$ and $H^p_{dF}(\mathcal{E}_x[t,s]) \cong 0$ for p > 0.

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Reduction to flatness along *x*-axis

Proposition

 $\omega = d(x + i\phi(t))$. If $\eta \in Z^{p}_{\omega}(\mathcal{E}^{n+1})$, then there exists $u \in \Lambda^{p-1}(\mathcal{E}^{n+1})$ such that

$$\eta - du \in Z^p_{\omega}(\mathcal{F}_0(\{t=0\}))$$

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$d\eta \wedge \omega = 0 \implies d\eta = \eta_1 \wedge \omega$ for some $\eta_1 \in \Lambda^p(\mathcal{E}^{n+1})$ $d^2\eta = d\eta_1 \wedge \omega = 0 \implies d\eta_1 = \eta_2 \wedge \omega$ for some $\eta_2 \in \Lambda^p(\mathcal{E}^{n+1})$ The continuation gives rise to a (Godbillion-Vey) sequence $\{\eta_j\}_j \subset \Lambda^p(\mathcal{E}^{n+1})$ with $d\eta_j = \eta_{j+1} \wedge \omega$. (set $\eta = \eta_0$). Let α be the formal *p*-form in (x, t, s):

$$\alpha = \sum_{j \ge 0} \eta_j \, \frac{(is)^j}{j!}$$

so that $oldsymbol{d} lpha = \left(\sum \eta_{j+1} rac{(is)^j}{j!}
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 $d\eta \wedge \omega = 0 \implies d\eta = \eta_1 \wedge \omega \text{ for some } \eta_1 \in \Lambda^p(\mathcal{E}^{n+1})$ $d^2\eta = d\eta_1 \wedge \omega = 0 \implies d\eta_1 = \eta_2 \wedge \omega \text{ for some } \eta_2 \in \Lambda^p(\mathcal{E}^{n+1})$ The continuation gives rise to a (Godbillion-Vey) sequence $\{\eta_j\}_j \subset \Lambda^p(\mathcal{E}^{n+1}) \text{ with } d\eta_j = \eta_{j+1} \wedge \omega. \text{ (set } \eta = \eta_0).$ Let α be the formal *p*-form in (x, t, s):

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Consider formal expansions w.r.t. *t* of η_j 's and w.r.t. (t, s) of α .

 $T_t \eta_j = \sum_r \eta_j^r t^r \text{ and } T_{t,s} \alpha = \sum_{j,r} \eta_j^r \frac{t^r (is)^j}{j!}$ with $\eta_j^r = \eta_j^r(x) \in \Lambda^p(\mathcal{E}_x)$. $d\alpha \wedge dF = 0 \implies T_{t,s} \alpha \in Z_{dF}^p(\mathcal{E}_x[t,s])$ so (from previous proposition) $\exists \widehat{\beta} \in \Lambda^{p-1}(\mathcal{E}_x[t,s])$ such that $(T_{t,s} \alpha - d\widehat{\beta}) \wedge dF = 0$. Let $\beta \in \Lambda^{p-1}(\mathcal{E}^{n+2})$ s.t. $T_{t,s}\beta = \widehat{\beta}$ (Borel). Let β_0 and $\eta = \eta_0$ be the pullbacks of β and α to s = 0. Then $T_t(\eta - d\beta_0) \wedge d(x + i\phi(t)) = 0$. i.e.

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$$\begin{split} T_{t}\eta_{j} &= \sum_{r} \eta_{j}^{r} t^{r} \text{ and } T_{t,s}\alpha = \sum_{j,r} \eta_{j}^{r} \frac{t^{r} (is)^{j}}{j!} \\ \text{with } \eta_{j}^{r} &= \eta_{j}^{r}(x) \in \Lambda^{p}(\mathcal{E}_{x}). \\ d\alpha \wedge dF &= 0 \implies T_{t,s}\alpha \in Z_{dF}^{p}(\mathcal{E}_{x}[t,s]) \\ \text{so (from previous proposition) } \exists \widehat{\beta} \in \Lambda^{p-1}(\mathcal{E}_{x}[t,s]) \text{ such that } \\ (T_{t,s}\alpha - d\widehat{\beta}) \wedge dF &= 0. \\ \text{Let } \beta \in \Lambda^{p-1}(\mathcal{E}^{n+2}) \text{ s.t. } T_{t,s}\beta = \widehat{\beta} \text{ (Borel).} \\ \text{Let } \beta_{0} \text{ and } \eta = \eta_{0} \text{ be the pullbacks of } \beta \text{ and } \alpha \text{ to } s = 0. \text{ Then } \\ T_{t}(\eta - d\beta_{0}) \wedge d(x + i\phi(t)) = 0. \text{ i.e.} \end{split}$$

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 $T_t \eta_j = \sum_r \eta_j^r t^r$ and $T_{t,s} \alpha = \sum_{j,r} \eta_j^r \frac{t^r (is)^j}{j!}$ with $\eta_i^r = \eta_i^r(x) \in \Lambda^p(\mathcal{E}_x)$. $d\alpha \wedge dF = 0 \implies T_{t,s}\alpha \in \mathbb{Z}^p_{\mathcal{A}F}(\mathcal{E}_x[t,s])$ so (from previous proposition) $\exists \hat{\beta} \in \Lambda^{p-1}(\mathcal{E}_x[t, s])$ such that

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Let $\beta \in \Lambda^{p-1}(\mathcal{E}^{n+2})$ s.t. $T_{t,s}\beta = \widehat{\beta}$ (Borel).
Let β_0 and $\eta = \eta_0$ be the pullbacks of β and α to $s = 0$. Then
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Consider formal expansions w.r.t. *t* of η_j 's and w.r.t. (t, s) of α .

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$$T_t \eta_j = \sum_r \eta_j^r t^r \text{ and } T_{t,s} \alpha = \sum_{j,r} \eta_j^r \frac{t^r (is)^j}{j!}$$

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$\phi(t) \in \mathcal{E}^n$ is \mathbb{R} -valued with an algebraically isolated singularity at 0.

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$$\begin{split} S^{n-1}_{\epsilon} &= \{ x \in \mathbb{R}^n : \ |x| = \epsilon \} \\ B^n_{\epsilon} &= \{ x \in \mathbb{R}^n : \ |x| < \epsilon \} . \\ V_{\epsilon} &= \phi^{-1}(0) \cap B^n_{\epsilon}, \\ Y_{\epsilon,\delta} &= |\phi|^{-1}(\delta) \cap B^n_{\epsilon} \\ U_{\epsilon,\delta} &= \phi^{-1}((-\delta, \ \delta)) \cap B^n_{\epsilon} \end{split}$$

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Theorem (C. A. Roche 1985)

For $\epsilon > 0$ small enough, there exists $\delta > 0$ s.t.

• S_{ϵ}^{n-1} is transversal to hypersurfaces $\phi^{-1}(t)$ for $|t| < \delta$.

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- V_{ϵ} is homeomorphic to a cone with vertex 0 and base $\overline{V_{\epsilon}} \cap S_{\epsilon}^{n-1}$.
- The restriction of |φ| to U\V_ε is a fibration with base (0, δ) and fiber Y_{ε,δ}

• $|\phi|: U_{\epsilon,\delta} \setminus V_{\epsilon} \longrightarrow (0, \delta)$ has a C^{∞} trivialization $\Psi: U_{\epsilon,\delta} \setminus V_{\epsilon} \longrightarrow Y_{\epsilon,\delta} \times (0, \delta)$ such that Ψ^* induces an isomorphism between $\mathcal{F}(Y_{\epsilon,\delta} \times \{0\}) \wedge^{\bullet}(C^{\infty}(Y_{\epsilon,\delta} \times (-\delta, \delta)))$ and $\mathcal{F}(V_{\epsilon}) \wedge^{\bullet}(C^{\infty}(U_{\epsilon,\delta})$

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M: smooth manifold of dimension m

 $\begin{aligned} R^+(\epsilon,\delta) &= \{(x,y) \in \mathbb{R}^2 : |x| < \epsilon, \ 0 \le y < \delta\} \\ Y &= \{(p,x,y) \in M \times R^+(\epsilon,\delta) : |y| = 0\} \\ \mathcal{F}(Y) \text{: germs of flat functions along } Y \text{ in } C^{\infty}(M \times \mathbb{R}^+(\epsilon,\delta)). \\ \Lambda^{\bullet}_{d(x+iy)}(\mathcal{F}(Y)) &= \frac{\Lambda^{\bullet}(\mathcal{F}(Y))}{\Lambda^{\bullet-1}(\mathcal{F}(Y)) \wedge d(x+iy)} \end{aligned}$

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Proposition

 $H^p_{dz}(\mathcal{F}(Y)) \cong H^{p-1}(M) \times \mathcal{S}$

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Proposition

 $H^{p}_{dz}(\mathcal{F}(Y)) \cong H^{p-1}(M) \times S$

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Proof

 $\eta = \eta^0 + \eta^1 \wedge d\overline{z} \mod(dz)$: p-form on $M \times \mathbb{R}^+(\epsilon, \delta)$ with η^0

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Proof

 $\eta = \eta^0 + \eta^1 \wedge d\overline{z} \mod(dz)$: p-form on $M \times \mathbb{R}^+(\epsilon, \delta)$ with η^0 and η^1 , p and (p-1) forms on M depending on parameter z.

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Proof

 $\eta = \eta^0 + \eta^1 \wedge d\overline{z} \mod(dz)$: p-form on $M \times \mathbb{R}^+(\epsilon, \delta)$ with η^0 and η^1 , p and (p-1) forms on M depending on parameter z. $d\eta = d_0\eta^0 + (d_0\eta^1 + (-1)^p\eta_{\overline{z}}^0) \wedge d\overline{z} \mod(dz)$

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Proof

 $\eta = \eta^0 + \eta^1 \wedge d\overline{z} \mod(dz)$: p-form on $M \times \mathbb{R}^+(\epsilon, \delta)$ with η^0 and η^1 , p and (p-1) forms on M depending on parameter z. $d\eta = d_0\eta^0 + (d_0\eta^1 + (-1)^p\eta_{\overline{z}}^0) \wedge d\overline{z} \mod(dz)$ If η is *dz*-closed, then $d_0\eta^0 = 0$ and $d_0\eta^1 + (-1)^p \eta_{\bar{z}}^0 = 0$.

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$$\frac{-1}{2\pi i}\int_{\mathbb{C}}\frac{g^{\nu}(\zeta)}{\zeta-z}\,d\zeta\wedge d\overline{\zeta}\,-H^{\nu}(z)$$

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is flat along y = 0. i.e. iff $[g^{\nu}] = 0 \in S$.

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is an isomorphism.

Let $W = (-a, a) \times U_{\epsilon, \delta} \subset \mathbb{R} \times \mathbb{R}^n$

 $W \setminus \mathbb{R} \times V = W_1 \cup \cdots \cup W_N.$ $L = |\phi|^{-1}(\delta), L_i \text{ connected component of } L \text{ such that}$ $(-a, a) \times L_i \subset W_i.$ $C_0^{\infty}(W_i)$: space of germs at 0 of C^{∞} functions with support $\subset \overline{W_i}.$

Proposition

For
$$p > 1$$
, $H^p_{\omega}(\mathcal{E}^{n+1}) \cong \bigoplus_{i=1}^N H^p_{\omega}(C_0^{\infty}(W_i))$

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Proposition

For
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First if $\eta \in Z_{\omega}^{p}(\mathcal{E}^{n+1})$, we can prove that $\eta = du + \eta^{0} \mod(\omega) \quad \text{with } \eta^{0} \in Z_{\omega}^{p}(\mathcal{F}_{0}(\mathbb{R} \times V))$ Define $\eta_{i}^{0} = \begin{cases} \eta^{0} & \text{in } W_{i} \\ 0 & \text{in } \mathbb{R}^{n+1} \setminus W_{i} \end{cases}$ so $\eta^{0} = \sum_{i} \eta_{i}^{0}$. Second we prove that $\eta_{i}^{0} \in B_{\omega}^{p}(\mathcal{E}^{n+1}) \implies \eta_{i}^{0} \in B_{\omega}^{p}(\mathcal{C}_{0}^{\infty}(\mathbb{R}))$

The map $[\eta] \longrightarrow \sum_{i} [\eta_{i}^{0}]$ gives an isomorphism

$$H^p_{\omega}(\mathcal{E}^{n+1}) \longrightarrow \bigoplus_i H^p_{\omega}(C_0^{\infty}(W_i))$$

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Second we prove that $\eta_{i}^{0} \in B_{\omega}^{p}(\mathcal{E}^{n+1}) \Longrightarrow \eta_{i}^{0} \in B_{\omega}^{p}(C_{0}^{\infty}(W_{i}))$
The map $[\eta] \longrightarrow \sum_{i} [\eta_{i}^{0}]$ gives an isomorphism

$$H^p_{\omega}(\mathcal{E}^{n+1}) \longrightarrow \bigoplus_i H^p_{\omega}(C_0^{\infty}(W_i))$$

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First if
$$\eta \in Z_{\omega}^{p}(\mathcal{E}^{n+1})$$
, we can prove that
 $\eta = du + \eta^{0} \mod(\omega)$ with $\eta^{0} \in Z_{\omega}^{p}(\mathcal{F}_{0}(\mathbb{R} \times V))$
Define $\eta_{i}^{0} = \begin{cases} \eta^{0} & \text{in } W_{i} \\ 0 & \text{in } \mathbb{R}^{n+1} \setminus W_{i} \end{cases}$
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$$H^p_\omega(\mathcal{E}^{n+1}) \longrightarrow \bigoplus_i H^p_\omega(C_0^\infty(W_i))$$

Proposition

$$H^p_\omega(C_0^\infty(W_i)) \cong H^{p-1}(L_i) \times S$$

Proof. Roche's Theorem induces an isomorphism

$$\begin{split} \Psi^*: \ \mathcal{F}((-a, \ a) \times L_i \times 0) \Lambda^{\bullet}(C^{\infty}((-a, \ a) \times L_i \times [0, \ \delta)) \\ \longrightarrow \ \Lambda^{\bullet}(C^{\infty}_0(W_i)) \end{split}$$

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which in turn induces an isomorphism

 $\mathrm{H}^{p-1}(L_i) \times \mathcal{S} \cong \mathrm{H}^p_{d(x+iy)}(\mathcal{F}(L_i \times R^+(\epsilon, \delta))) \cong \mathrm{H}^p_{d(x+i\phi(t))}(C_0^{\infty}(W_i))$

Proposition

 $H^p_\omega(C_0^\infty(W_i)) \cong H^{p-1}(L_i) imes S$

Proof. Roche's Theorem induces an isomorphism

$$\Psi^*: \ \mathcal{F}((-a, \ a) \times L_i \times 0) \wedge^{\bullet}(C^{\infty}((-a, \ a) \times L_i \times [0, \ \delta)) \\ \longrightarrow \ \wedge^{\bullet}(C^{\infty}_0(W_i))$$

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