# 1 1.4 Complex Numbers

**Definition 1.1.** The imaginary unit $i$ is defined as

$$i = \sqrt{-1}, \quad \text{where } i^2 = -1.$$

**Example 1.1.** $\sqrt{-25} = \sqrt{-1} * \sqrt{25} = i * 5 = 5i$

**Example 1.2.** $(5i)^2 = 5^2 * i^2 = 25 * (-1) = -25$

**Exercise 1.1.** Simplify the following.

1. $(5 - 11i) + (7 + 4i)$

2. $(-5 + i) - (-11 - 6i)$

3. $4i(3 - 5i)$

4. $(7 - 3i)(-2 - 5i)$

5. $7i(2 - 9i)$

6. $(5 + 4i)(6 - 7i)$

To divide two complex numbers we have to multiply the denominator by its conjugate to eliminate $i$.

**Definition 1.2.** Given a complex number $a + bi$ and $a - bi$, the complex conjugate is $a - bi$ and $a + bi$, respectively.

**Example 1.3.**

$$\frac{3i}{4+i} = \frac{3i}{4+i} * \frac{4-i}{4-i} = \frac{3i(4-i)}{(4+i)(4-i)} = \frac{12i - 3i^2}{16 - 4i + 4i - i^2} = \frac{12i - 3 * (-1)}{16 - (-1)} = \frac{3 + 12i}{17} = \frac{3}{17} + \frac{12}{17}i$$

Let's practice this!

**Exercise 1.2.** Divide and express the result in standard form.

1. $\dfrac{5i}{7+i}$

2. $\dfrac{7+4i}{2-5i}$

Let's look at Square Root. Squaring 5 or -5 will give you 25, i.e., $5^2 = (-5)^2 = 25$. Therefore, every number has two square roots. For example $\sqrt{36} = 6$ and $\sqrt{36} = -6$. To make sense in this, we will call the positive number to be **the (principal) square root**.

**Exercise 1.3.** Find a square root for the following numbers: 4, 9, 36, 81.

**Exercise 1.4.** Find the (principal) square root for the following numbers: 4, 9, 36, 81.

Are your answers for the two exercises above the same? Do they have to be the same?

Similarly to positive numbers, we have two square roots for negative numbers, i.e., $\sqrt{-25} = 5i$ and $\sqrt{-25} = -5i$. The **principal square root** of a negative number is the positive complex number.

**Exercise 1.5.** Find a square root for the following numbers: -25, -49, -64.

**Exercise 1.6.** Find the principal square root for the following numbers: -25, -49, -64.

Are your answers for the two exercises above the same? Do they have to be the same?

**Exercise 1.7.** Perform the indicated operations and write the result in standard form (a+bi). Use the principal square roots when needed.

1. $\sqrt{-18} - \sqrt{-8}$

2. $(-1 + \sqrt{-5})^2$

3. $\dfrac{-25 + \sqrt{-50}}{15}$