

## TRIGONOMETRIC IDENTITIES

- **Reciprocal identities**

$$\begin{aligned}\sin u &= \frac{1}{\csc u} & \cos u &= \frac{1}{\sec u} \\ \tan u &= \frac{1}{\cot u} & \cot u &= \frac{1}{\tan u} \\ \csc u &= \frac{1}{\sin u} & \sec u &= \frac{1}{\cos u}\end{aligned}$$

- **Pythagorean Identities**

$$\begin{aligned}\sin^2 u + \cos^2 u &= 1 \\ 1 + \tan^2 u &= \sec^2 u \\ 1 + \cot^2 u &= \csc^2 u\end{aligned}$$

- **Quotient Identities**

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

- **Co-Function Identities**

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \quad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\csc\left(\frac{\pi}{2} - u\right) = \sec u \quad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

- **Parity Identities (Even & Odd)**

$$\begin{aligned}\sin(-u) &= -\sin u & \cos(-u) &= \cos u \\ \tan(-u) &= -\tan u & \cot(-u) &= -\cot u \\ \csc(-u) &= -\csc u & \sec(-u) &= \sec u\end{aligned}$$

- **Sum & Difference Formulas**

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$

- **Double Angle Formulas**

$$\begin{aligned}\sin(2u) &= 2 \sin u \cos u \\ \cos(2u) &= \cos^2 u - \sin^2 u \\ &= 2 \cos^2 u - 1 \\ &= 1 - 2 \sin^2 u \\ \tan(2u) &= \frac{2 \tan u}{1 - \tan^2 u}\end{aligned}$$

- **Power-Reducing/Half Angle Formulas**

$$\sin\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1}{2}(1 - \cos u)}$$

$$\cos\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1}{2}(1 + \cos u)}, \quad \tan\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 - \cos u}{1 + \cos u}}$$

- **Sum-to-Product Formulas**

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

- **Product-to-Sum Formulas**

$$\sin u \sin v = \frac{1}{2} [\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u+v) - \sin(u-v)]$$