MAC2233 - Review for exam 2

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

$C(x)$ is the total cost of producing x units of a particular commodity and $p(x)$ is the unit price at which all x units will be sold. Assume $p(x)$ and $C(x)$ are in dollars. Find the marginal cost and the marginal revenue.

1) $C(x) = \frac{1}{4}x^2 + 5x + 53; p(x) = \frac{1}{3}(36 - x)$

$C(x)$ is the total cost of producing x units of a particular commodity. Assume $C(x)$ is in dollars. Use marginal cost to estimate the cost of producing the 21st unit. What is the actual cost of producing the 21st unit?

2) $C(x) = \frac{1}{4}x^2 + 4x + 65$

Use increments to make the required estimate.

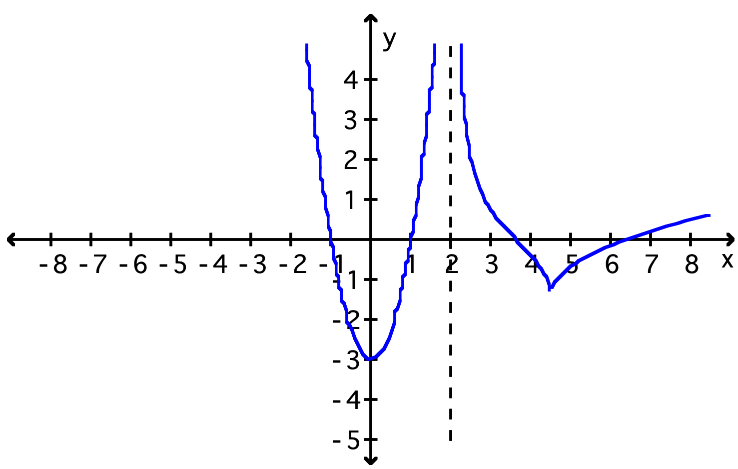
3) Estimate how much the function $f(x) = x^2 - 3x + 3$ will change as x increases from 5 to 5.3.

Solve the problem.

4) A manufacturer's total cost is $C(q) = 0.2q^3 - 0.2q^2 + 500q + 230$ when q thousand units are produced. Currently, 4000 units ($q = 4$) are being produced and the manufacturer is planning to increase the level of production to 4100. Use marginal analysis to estimate how this change will affect total cost.

Specify the intervals on which the derivative of the given function is positive and those on which it is negative.

5)



Find the intervals of increase and decrease for the given function.

6) $f(x) = x^2 + 4x + 3$

7) $f(x) = x^3 - 27x - 7$

8) $f(t) = \frac{1}{49 - t^2}$

9) $f(t) = \frac{t}{(t + 3)^2}$

Determine the critical numbers of the given function and classify each critical point as a relative maximum, a relative minimum, or neither.

10) $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$

11) $f(t) = \frac{t}{t^2 + 11}$

12) $g(x) = 4 - \frac{2}{x} + \frac{9}{x^2}$

Use calculus to sketch the graph of the given function.

13) $f(x) = x^3 - 3x^2$

14) $f(x) = 3x^4 + 8x^3 + 6x^2 - 3$

15) $f(x) = x^3(x + 4)^2$

16) $g(t) = \frac{t}{t^2 + 5}$

The derivative of a function $f(x)$ is given. In each case, find the critical numbers of $f(x)$ and classify each as corresponding to a relative maximum, a relative minimum, or neither.

17) $f'(x) = x^2(9 - x^2)$

Determine where the graph of the given function is concave upward and concave downward. Find the coordinates of all inflection points.

18) $f(x) = x^3 + 6x^2 + x + 9$

19) $g(t) = t^2 - \frac{27}{t}$

Use first and second derivative information to sketch the graph of the function.

20) $f(x) = \frac{1}{3}x^3 - 9x + 3$

21) $f(x) = 2x^5 - 10x - 7$

22) $g(x) = \sqrt{x^2 + 9}$

23) $f(x) = \dfrac{2}{x^2 + x + 7}$

Use the second derivative test to find the relative maxima and minima of the given function.

24) $f(x) = x^4 - 32x^2 + 7$

25) $f(x) = 2x + 9 + \dfrac{8}{x}$

26) $h(x) = \dfrac{5}{1 + x^2}$

The second derivative $f''(x)$ of a function is given. In each case, use this information to determine where the graph of $f(x)$ is concave upward and concave downward and find all values of x for which an inflection point occurs. [You are not required to find $f(x)$ or the y coordinates of the inflection points.]

27) $f''(x) = x^2(x - 7)(x - 5)$

Solve the problem.

28) Sketch the graph of a function that has all the following properties:

a. $f'(x) > 0$ when $x < -1$ and when $x > 1$
b. $f'(x) < 0$ when $-1 < x < 1$
c. $f''(x) < 0$ when $x < 0$
d. $f''(x) > 0$ when $x > 0$

Find all vertical and horizontal asymptotes of the graph of the given function.

29) $f(x) = \dfrac{6x - 9}{x + 2}$

30) $f(t) = \dfrac{t + 5}{t^2}$

Sketch the graph of the given function.

31) $f(x) = x^4 + 4x^3 + 4x^2 - 1$

32) $f(x) = x - \dfrac{1}{x}$

33) $f(x) = \dfrac{x^2 - 9}{x^2 + 1}$

Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

34) $f(x) = x^3 - 6x^2 + 7; -1 \le x \le 6$

35) $f(x) = x^5 - 20x^4 + 8; 0 \le x \le 17$

36) $f(t) = \dfrac{t^2}{t - 8}; -2 \le t \le -\dfrac{1}{4}$

37) $f(x) = \dfrac{1}{x + 12}; x \ge 0$

1) marginal cost = $\frac{1}{2}x + 5$;

marginal revenue = $12 - \frac{2x}{3}$

2) estimated cost = \$14.00; actual cost = \$14.25

3) 2.1

4) The approximate change in cost will be \$50.80.

5) $f'(x) > 0$ for $0 < x < 2$ and $x > 4.5$; $f'(x) < 0$ for $x < 0$ and $2 < x < 4.5$

6) $f(x)$ is increasing for $x > -2$; $f(x)$ is decreasing for $x < -2$.

7) $f(x)$ is increasing for $x < -3$ and $x > 3$; $f(x)$ is decreasing for $-3 < x < 3$.

8) $f(t)$ is increasing for $0 < t < 7$ and $t > 7$; $f(t)$ is decreasing for $t < -7$ and $-7 < t < 0$.

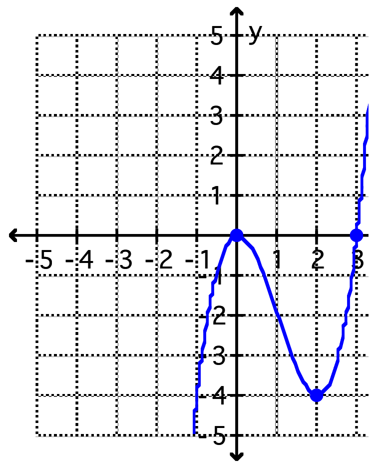
9) $f(t)$ is increasing on $-3 < t < 3$; $f(t)$ is decreasing on $t < -3$ and $t > 3$.

10) $x = 0, 1$; (0, 1) relative minimum; (1, 2) neither

11) $t = -\sqrt{11}, \sqrt{11}$; $\left(-\sqrt{11}, -\frac{\sqrt{11}}{22}\right)$ relative minimum; $\left(\sqrt{11}, \frac{\sqrt{11}}{22}\right)$ relative maximum

12) $x = 9$; $\left(9, \frac{35}{9}\right)$ relative minimum

13)



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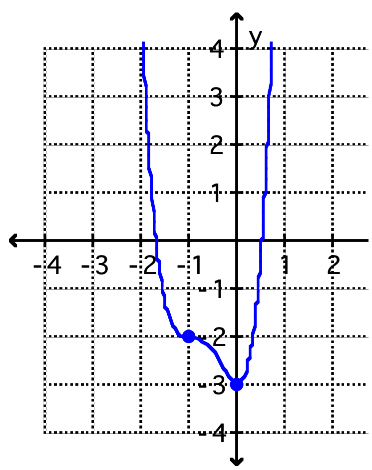

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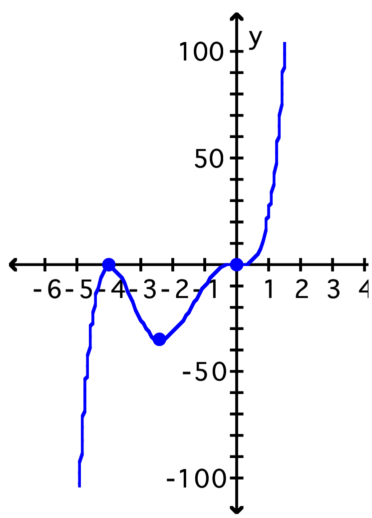

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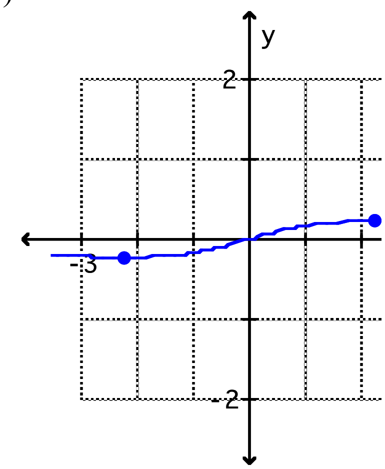


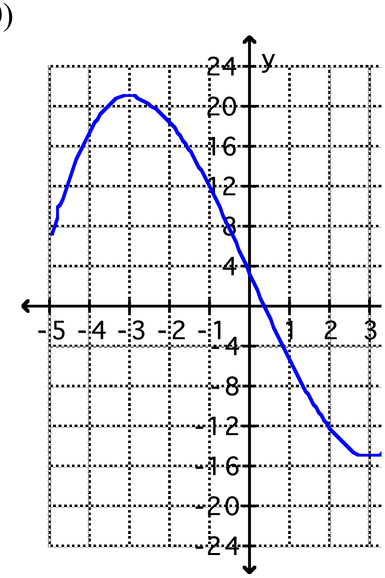
17)

Critical Numbers	Cla
-3	Relat
0	
3	Relati

18) Concave upward for $x > -2$; concave downward for $x < -2$; inflection at (-2, 11)

19) upward for $t < 0$ and $t > 3$; downward for $0 < t < 3$; inflection at (3, 0)

20)



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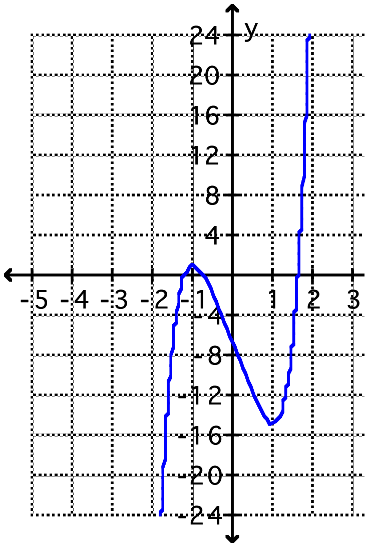

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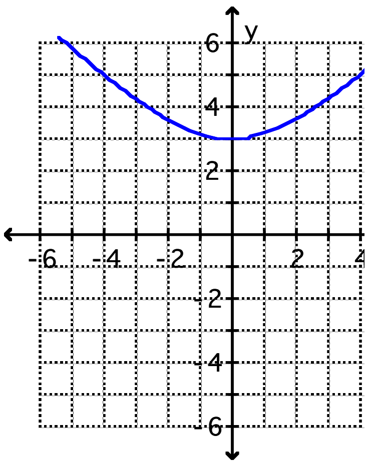

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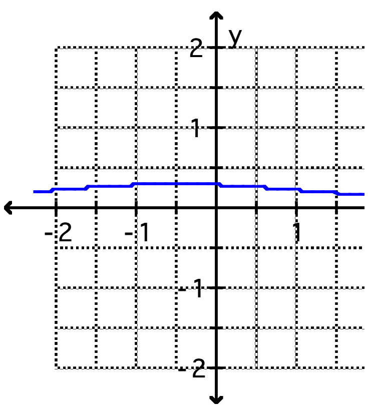


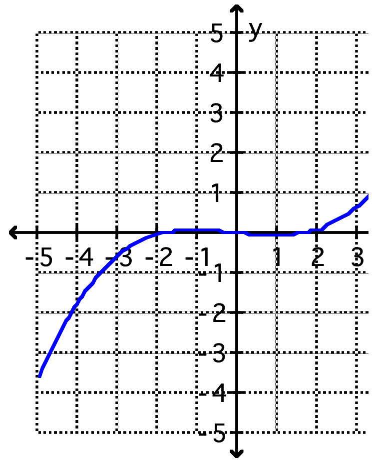
24) min at (-4, -249); max at (0, 7); min at (4, -249)

25) max at (-2, 1); min at (2, 17)

26) max at (0, 5)

27) Concave upward for $x < 0$, for $0 < x < 5$, and for $x > 7$; concave downward for $5 < x < 7$; inflection points at $x = 5, 7$

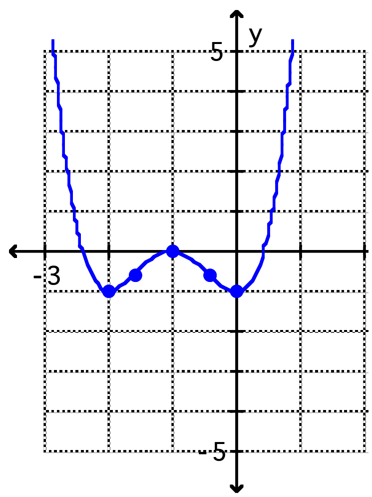
28)



29) Vertical asymptote, $x = -2$; horizontal asymptote, $y = 6$

30) Vertical asymptote, $t = 0$; horizontal asymptote, $y = 0$

31)



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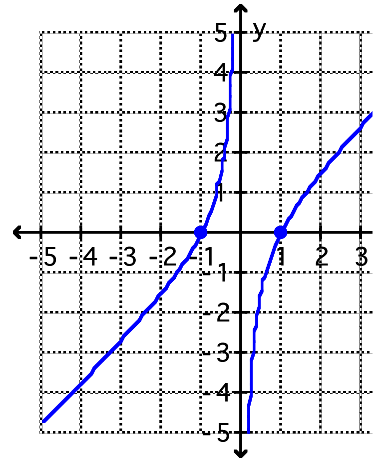

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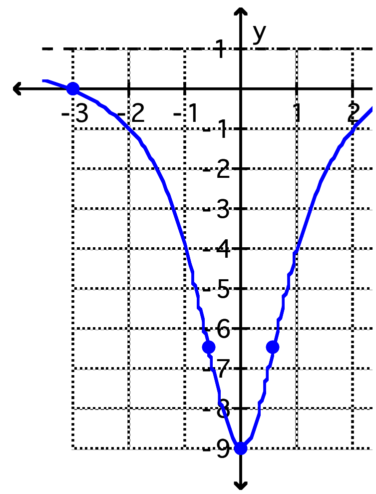

34) min = -25; max = 7

35) min = -262,136; max = 8

36) min = $-\frac{2}{5}$; max = $-\frac{1}{132}$

37) no min; max = $\frac{1}{12}$