MAC2233 - Review for 3.4, 4.1 - 4.4

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

1) $f(x) = x^3 - 6x^2 + 7; -1 \le x \le 6$

2) $f(x) = x^5 - 20x^4 + 8; 0 \le x \le 17$

3) $f(t) = \dfrac{t^2}{t - 8}; -2 \le t \le -\dfrac{1}{4}$

4) $f(x) = \dfrac{1}{x + 12}; x \ge 0$

Compute the elasticity of demand for the given demand function $D(p)$ and determine whether the demand is elastic, inelastic, or of unit elasticity at the indicated price p.

5) $D(p) = 150 - p^2; p = 9$

6) $D(p) = \dfrac{2300}{p^2}; p = 8$

Solve the problem.

7) When a particular commodity is priced at p dollars per unit, consumers demand q units, where p and q are related by the equation $q^2 + 5pq = 24$.

a. Find the elasticity of demand for this commodity.
b. For a unit price of \$2, is the demand elastic, inelastic, or of unit elasticity?

8) An airline determines that when a round-trip ticket between Los Angeles and San Francisco costs p dollars ($0 \le p \le 120$), the daily demand for tickets is $q = 432 - 0.03p^2$.
a. Find the elasticity of demand. Determine the values of p for which the demand is elastic, inelastic, and of unit elasticity.
b. Interpret the results of part (**a**) in terms of the behavior of the total revenue as a function of unit price p.

9) A store has been selling a popular computer game at the price of \$44 per unit, and at this price, players have been buying 72 units per month. The owner of the store wishes to raise the price of the game and estimates that for each \$1 increase in price, three fewer units will be sold each month. If each unit costs the store \$26, at what price should the game be sold to maximize profit?

10) It is estimated that the cost of constructing an office building that is n floors high is $C(n) = 3n^2 + 300n + 700$ thousand dollars. How many floors should the building have to minimize the average cost per floor? (Remember that your answer should be a whole number.)

11) Find the present value of $17,000 over a term of 4 years at an annual interest rate of 8% if interest is compounded:
a. Annually
b. Quarterly

12) Find the present value of $16,000 over a term of 8 years at an annual interest rate of 7% if interest is compounded continuously.

13) Bob and Alice want to remodel their bathroom in 3 years. They estimate the job will cost $29,000. How much must they invest now at an annual interest rate of 6% compounded quarterly to achieve their goal?

14) The Morenos invest $9000 in an account that grows to $11,000 in 4 years. What is the annual interest rate r if interest is compounded
a. Quarterly **b.** Continuously

15) How quickly will money double if it is invested at an annual interest rate of 10% compounded continuously?

Differentiate the given function.

16) $f(x) = 9e^{9x + 7}$

17) $f(x) = \dfrac{3e^x}{2x}$

18) $f(x) = xe^{-x^3}$

19) $f(x) = x^9 \ln x$

20) $f(x) = e^{-5x} + x^4$

21) $f(x) = \ln(9x^3 + 8x - 5)$

22) $f(t) = \sqrt{6\ln t + 7t}$

23) $f(x) = x^6 5^{x^5}$

Find the largest and smallest values of the given function over the prescribed closed, bounded interval.

24) $F(x) = 3e^{(3x^2 - 6x)}$ for $0 \le x \le 2$

25) $g(x) = \dfrac{e^x}{5x + 1}$ for $0 \le x \le 1$

Find the second derivative of the given function.

26) $f(x) = 4e^{2x} + 9e^{-x}$

Find an equation for the tangent line to $y = f(x)$ at the specified point.

27) $f(x) = -5xe^{-x}$; where $x = 0$

28) $f(x) = x - 5\ln x$; where $x = e$

Use logarithmic differentiation to find the derivative $f'(x)$.

29) $f(x) = (3x + 4)^2(x - 4x^2)^{1/2}$

30) $f(x) = \sqrt[3]{\dfrac{2x + 1}{1 - 5x}}$

Solve the problem.

31) Paul Edwards owns an electronics firm. He determines that when he employs x thousand people, the profit will be P million dollars, where

$$P(x) = \frac{2}{5}\ln(5x + 2) + 3x - x^2$$

How many workers should Paul employ to maximize profit? What is the maximum profit?

1) min = -25; max = 7

2) min = -262,136; max = 8

3) min = $-\frac{2}{5}$; max = $-\frac{1}{132}$

4) no min; max = $\frac{1}{12}$

5) $E(p) = \frac{2p^2}{150 - p^2}$; $E(9) \approx$ 2.348, elastic

6) $E(p) = 2$; $E(8) = 2$, elastic

7) **a.** $E = \frac{5p}{2q + 5p}$

b. When $p = 2$, demand is inelastic.

8) **a.** $E(p) = \frac{0.02p^2}{144 - 0.01p^2}$

Demand is elastic when $p >$ 69.28, inelastic when $p <$ 69.28, and of unit elasticity when $p = 69.28$.

b. If a ticket costs more than \$69.28 then revenue is decreasing as the price increases. If the cost is less than \$69.28 then revenue increases with price. If the price equals \$69.28 then revenue is unaffected by a small change in price.

9) $47

10) 15 floors

11) **a.** $12,495.51

b. $12,383.58

12) ≈ $9139.35

13) ≈ $24,255.24

14) **a.** ≈ 5.048%

b. ≈ 5.017%

15) ≈ 6.93 years

16) $f'(x) = 81e^{9x + 7}$

17) $f'(x) = \frac{3e^x(x - 1)}{2x^2}$

18) $f'(x) = e^{-x^3}(1 - 3x^3)$

19) $f'(x) = x^8(9\ln x + 1)$

20) $f'(x) = -5e^{-5x} + 4x^3$

21) $f'(x) = \frac{27x^2 + 8}{9x^3 + 8x - 5}$

22) $f'(t) = \frac{7t + 6}{2t\sqrt{6\ln t + 7t}}$

23) $6x^5 5^{x^5} + 5x^{10}(\ln 5)5^{x^5}$

24) smallest: $3e^{-3}$; largest: 3

25) smallest: $\frac{1}{5}e^{4/5}$; largest:1

26) $f''(x) = 16e^{2x} + 9e^{-x}$

27) $y = -5x$

28) $y = (1 - \frac{5}{e})x$

29) $f'(x) = f(x)\left[\frac{6}{3x + 4} + \frac{1 - 8x}{2(x - 4x^2)}\right]$

30) $f'(x) = f(x)\left(\frac{1}{3}\right)\left(\frac{2}{2x + 1} + \frac{5}{1 - 5x}\right)$

31) 1600 people; ≈ $3,161,034