MAC2233 - Review for exam 2

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Differentiate the given function.

1) $y = \dfrac{x + 9}{x - 7}$

2) $f(t) = \dfrac{t}{t^2 + 6}$

3) $f(x) = \dfrac{x^2 + 5x + 3}{2x^2 + 3x + 4}$

Find an equation for the tangent line to the given curve at the point where $x = x_0$.

4) $y = \dfrac{x + 5}{2 - 3x}$; $x_0 = 0$

Use the chain rule to compute the derivative $\dfrac{dy}{dx}$ and simplify your answer.

5) $y = \dfrac{4}{u - 6}$; $u = x^2$

Use the chain rule to compute the derivative $\dfrac{dy}{dx}$ for the given value of x.

6) $y = \sqrt{u}$; $u = x^2 + 3x - 18$ for $x = 6$

Differentiate the given function and simplify your answer.

7) $f(x) = (8x - 5)^4$

8) $f(t) = \dfrac{1}{5t^2 - 8t + 9}$

9) $f(s) = \dfrac{1}{\sqrt{7s^3 + 3}}$

10) $f(x) = (x + 3)^3(2x - 1)^5$

Find an equation of the line that is tangent to the graph of f for the given value of x.

11) $f(x) = \sqrt{4x + 36}$; $x = 0$

Find all values of $x = c$ so that the tangent line to the graph of $f(x)$ at $(c, f(c))$ will be horizontal.

12) $f(x) = (x^2 + 5x)^4$

13) $f(x) = \dfrac{x}{(2x - 5)^2}$

Find the second derivative of the given function.

14) $f(t) = \dfrac{3}{4t + 5}$

Solve the problem.

15) When a certain commodity is sold for p dollars per unit, consumers will buy $D(p) = \dfrac{40{,}000}{p}$ units per month. It is estimated that t months from now, the price of the commodity will be $p(t) = 0.5t^{3/2} + 6$ dollars per unit. At what percentage rate will the monthly demand for the commodity be changing with respect to time 4 months from now?

16) At a certain factory, the total cost of manufacturing q units is $C(q) = 0.4q^2 + q + 900$ dollars. It has been determined that approximately $q(t) = t^2 + 80t$ units are manufactured during the first t hours of a production run. Compute the rate at which the total manufacturing cost is changing with respect to time 2 hour after production begins.

$C(x)$ is the total cost of producing x units of a particular commodity and $p(x)$ is the unit price at which all x units will be sold. Assume $p(x)$ and $C(x)$ are in dollars. Find the marginal cost and the marginal revenue.

17) $C(x) = \dfrac{1}{4}x^2 + 5x + 53; p(x) = \dfrac{1}{3}(36 - x)$

$C(x)$ is the total cost of producing x units of a particular commodity. Assume $C(x)$ is in dollars. Use marginal cost to estimate the cost of producing the 21st unit. What is the actual cost of producing the 21st unit?

18) $C(x) = \dfrac{1}{4}x^2 + 4x + 65$

Use increments to make the required estimate.

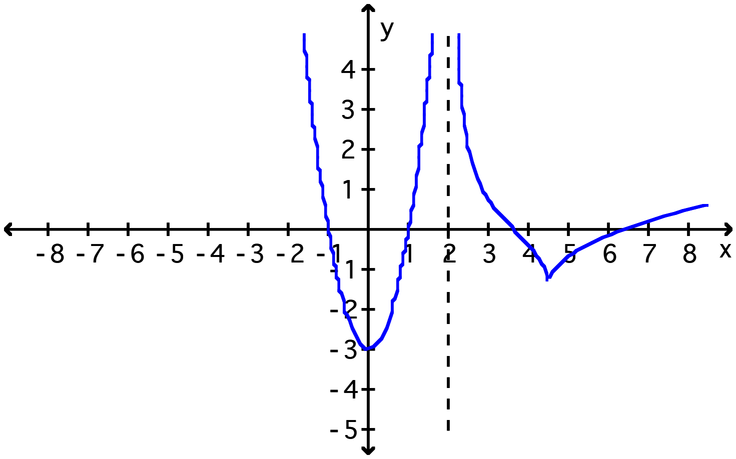
19) Estimate how much the function $f(x) = x^2 - 3x + 3$ will change as x increases from 5 to 5.3.

Solve the problem.

20) A manufacturer's total cost is $C(q) = 0.2q^3 - 0.2q^2 + 500q + 230$ when q thousand units are produced. Currently, 4000 units ($q = 4$) are being produced and the manufacturer is planning to increase the level of production to 4100. Use marginal analysis to estimate how this change will affect total cost.

Specify the intervals on which the derivative of the given function is positive and those on which it is negative.

21)



Find the intervals of increase and decrease for the given function.

22) $f(x) = x^2 + 4x + 3$

23) $f(x) = x^3 - 27x - 7$

24) $f(t) = \dfrac{1}{49 - t^2}$

25) $f(t) = \dfrac{t}{(t + 3)^2}$

Determine the critical numbers of the given function and classify each critical point as a relative maximum, a relative minimum, or neither.

26) $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$

27) $f(t) = \dfrac{t}{t^2 + 11}$

28) $g(x) = 4 - \dfrac{2}{x} + \dfrac{9}{x^2}$

Use calculus to sketch the graph of the given function.

29) $f(x) = x^3 - 3x^2$

30) $f(x) = 3x^4 + 8x^3 + 6x^2 - 3$

31) $f(x) = x^3(x + 4)^2$

32) $g(t) = \frac{t}{t^2 + 5}$

Determine where the graph of the given function is concave upward and concave downward. Find the coordinates of all inflection points.

33) $f(x) = x^3 + 6x^2 + x + 9$

34) $g(t) = t^2 - \frac{27}{t}$

Use first and second derivative information to sketch the graph of the function.

35) $f(x) = \frac{1}{3}x^3 - 9x + 3$

36) $f(x) = 2x^5 - 10x - 7$

37) $g(x) = \sqrt{x^2 + 9}$

Use the second derivative test to find the relative maxima and minima of the given function.

38) $f(x) = x^4 - 32x^2 + 7$

39) $f(x) = 2x + 9 + \frac{8}{x}$

40) $h(x) = \frac{5}{1 + x^2}$

Solve the problem.

41) Sketch the graph of a function that has all the following properties:

a. $f'(x) > 0$ when $x < -1$ and when $x > 1$

b. $f'(x) < 0$ when $-1 < x < 1$

c. $f''(x) < 0$ when $x < 0$

d. $f''(x) > 0$ when $x > 0$

Find all vertical and horizontal asymptotes of the graph of the given function.

42) $f(x) = \frac{6x - 9}{x + 2}$

43) $f(t) = \frac{t + 5}{t^2}$

Sketch the graph of the given function.

44) $f(x) = x^4 + 4x^3 + 4x^2 - 1$

45) $f(x) = x - \frac{1}{x}$

1) $\dfrac{-16}{(x-7)^2}$

2) $\dfrac{-t^2+6}{(t^2+6)^2}$

3) $\dfrac{-7x^2-4x+11}{(2x^2+3x+4)^2}$

4) $y = \dfrac{17}{4}x + \dfrac{5}{2}$

5) $\dfrac{-8x}{(x^2-6)^2}$

6) $\dfrac{5}{4}$

7) $f'(x) = 32(8x-5)^3$

8) $f'(t) = \dfrac{-2(5t-4)}{(5t^2-8t+9)^2}$

9) $f(s) = \dfrac{-21s^2}{2(7s^3+3)^{3/2}}$

10) $f'(x) = (x+3)^2(2x-1)^4(16x+27)$

11) $y = \dfrac{1}{3}x + 6$

12) $x = 0$; $x = -5$; $x = -\dfrac{5}{2}$

13) $x = -\dfrac{5}{2}$

14) $f''(t) = \dfrac{96}{(4t+5)^3}$

15) -15%

16) $11,104.80 per hour

17) marginal cost $= \dfrac{1}{2}x + 5$;

marginal revenue $= 12 - \dfrac{2x}{3}$

18) estimated cost = $14.00; actual cost = $14.25

19) 2.1

20) The approximate change in cost will be $50.80.

21) $f'(x) > 0$ for $0 < x < 2$ and $x > 4.5$; $f'(x) < 0$ for $x < 0$ and $2 < x < 4.5$

22) $f(x)$ is increasing for $x > -2$; $f(x)$ is decreasing for $x < -2$.

23) $f(x)$ is increasing for $x < -3$ and $x > 3$; $f(x)$ is decreasing for $-3 < x < 3$.

24) $f(t)$ is increasing for $0 < t < 7$ and $t > 7$; $f(t)$ is decreasing for $t < -7$ and $-7 < t < 0$.

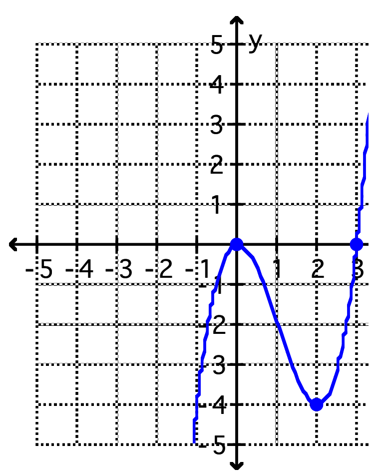
25) $f(t)$ is increasing on $-3 < t < 3$; $f(t)$ is decreasing on $t < -3$ and $t > 3$.

26) $x = 0, 1$; $(0, 1)$ relative minimum; $(1, 2)$ neither

27) $t = -\sqrt{11}, \sqrt{11}$; $\left(-\sqrt{11}, -\dfrac{\sqrt{11}}{22}\right)$ relative minimum; $\left(\sqrt{11}, \dfrac{\sqrt{11}}{22}\right)$ relative maximum

28) $x = 9$; $\left(9, \dfrac{35}{9}\right)$ relative minimum

29)



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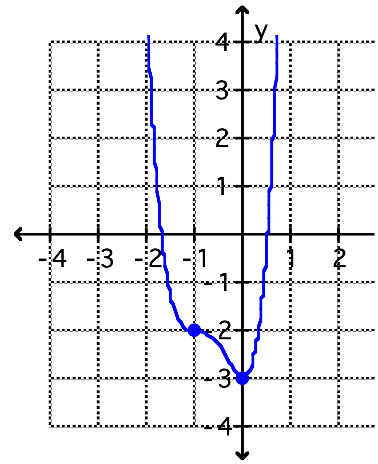

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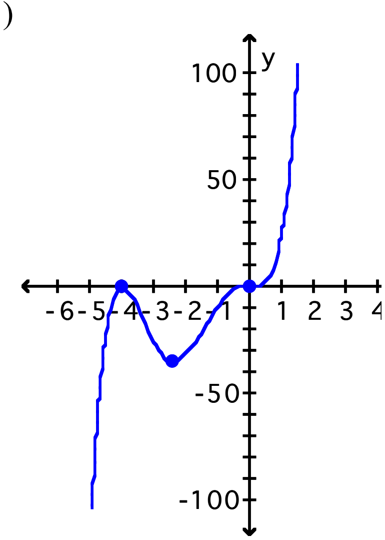

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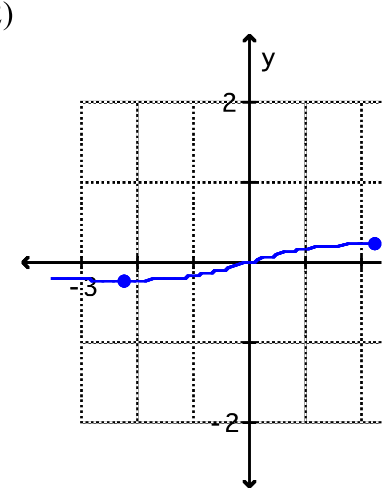
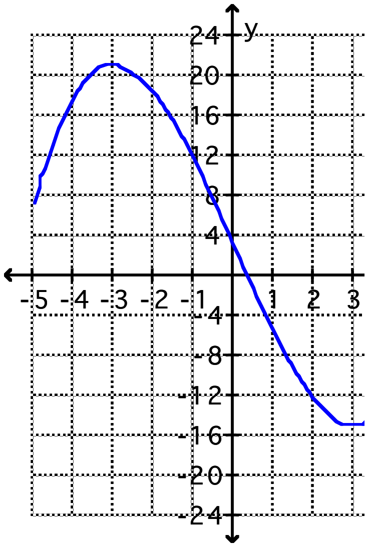


33) Concave upward for $x > -2$; concave downward for $x < -2$; inflection at $(-2, 11)$

34) upward for $t < 0$ and $t > 3$; downward for $0 < t < 3$; inflection at (3, 0)

35)



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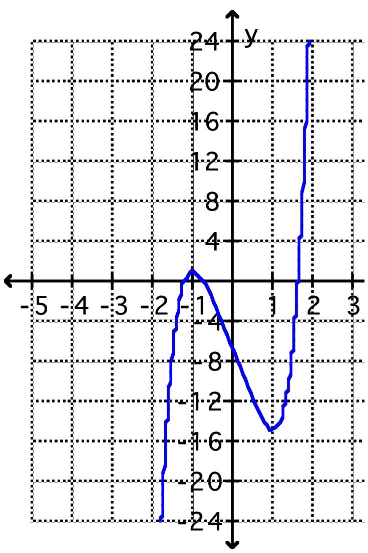

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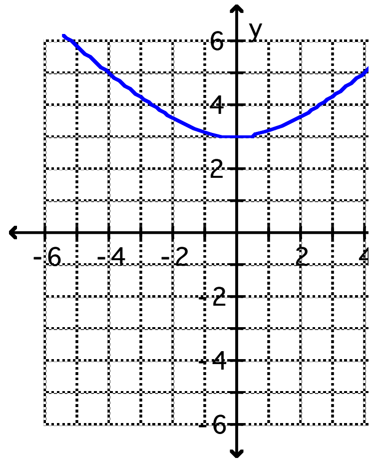
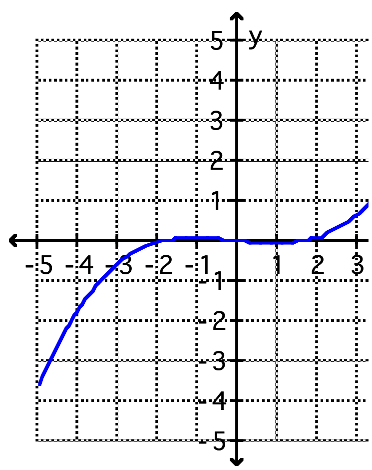


38) min at (-4, -249); max at (0, 7); min at (4, -249)

39) max at (-2, 1); min at (2, 17)

40) max at (0, 5)

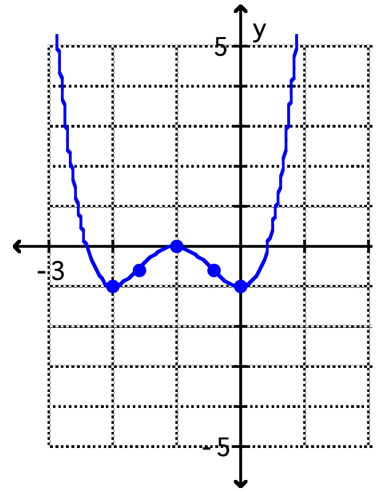
41)



42) Vertical asymptote, $x = -2$; horizontal asymptote, $y = 6$

43) Vertical asymptote, $t = 0$; horizontal asymptote, $y = 0$

44)



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