MAC 1105, Fall 2017.

# Exam #1

October 3, 2017

**Name** Key

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who uses a cell phone during the examination or if one is found within hands reach.
- Calculators are not allowed on this exam.
- The exam consists of two parts. Part I contains four multiple choice questions worth 5 points each. Part II contains 8 open ended questions.

## Part I

Choose your answer from five available choices. No partial credit will be given for wrong answers.

1. Simplify $\dfrac{\sqrt{12x^2}}{6x^2}$

$$\frac{\sqrt{12x^2}}{6x^2} = \frac{\sqrt{4}\cdot\sqrt{3}\cdot\sqrt{x^2}}{6x^2} = \frac{2\sqrt{3}\cdot x}{6x^2} = \frac{\sqrt{3}}{3x}$$

   (a) $\dfrac{2}{1}$

   (b) $\dfrac{\sqrt{3}}{3x}$

   (c) $\dfrac{\sqrt{12}}{6}$

   (d) $\sqrt{3}$

   (e) None of the above

2. Divide the following complex numbers and express the result in standard form, $a+bi$.

$$\frac{2i-3}{2+i}\cdot\frac{2-i}{2-i} = \frac{4i-2i^2-6+3i}{4-i^2}$$

$$= \frac{4i-2(-1)-6+3i}{4-(-1)} = \frac{4i+2-6+3i}{4+1}$$

$$= \frac{7i-4}{5} = \frac{-4}{5}+\frac{7}{5}i$$

   (a) $\dfrac{4-7i}{5}$

   (b) $-\frac{8}{5}+\frac{7}{5}i$

   (c) $-4+7i$

   (d) $-\frac{4}{5}+\frac{7}{5}i$

   (e) None of the above

3. Find the solution set for the equation

$$(x-1)^2 = -9$$

$$x-1 = \pm\sqrt{-9}$$

$$x-1 = \pm i\sqrt{9} = \pm i\cdot 3$$

$$x-1 = \pm 3i$$

$$x = 1 \pm 3i$$

   (a) $\{1+3i, 1-3i\}$

   (b) $\{3i+1, 3i-1\}$

   (c) $\{1+3i\}$

   (d) The solution set is empty.

   (e) None of the above

4. Determine the number and type of solutions for the following equation

$$x^2 - 3x + 5 = 0$$

discriminant:
$(-3)^2 - 4\cdot 1\cdot 5 = 9 - 20 = -11 < 0$

(a) One real solutions.
(b) Two real solution.
(c) Two complex solutions.
(d) Three radical solutions.
(e) None of the above.

## Part II

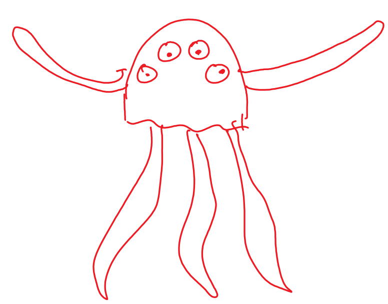
5. (10 points) Consider the points $(1, 6)$ and $(4, 2)$. Draw a squid or octopus that has

(a) The number of arms the same as the distance between the points.
(b) The number of eyes the same as the y-coordinate of the midpoint.

(a) distance $= \sqrt{(1-4)^2 + (6-2)^2} = \sqrt{9 + 16} = \sqrt{25} = \boxed{5}$

(b) midpoint $= \left(\frac{1+4}{2}, \frac{6+2}{2}\right) = \left(\frac{5}{2}, 4\right)$

y-coordinate $= \boxed{4}$



6. (10 points each) Solve for $x$ and include any complex solutions.

(a) $\sqrt{2x-1}+2=x$

$$-2 \quad -2$$

$$\sqrt{2x-1} = x-2 \quad ()^2$$

$$(\sqrt{2x-1})^2 = (x-2)^2$$

$$2x-1 = x^2-4x+4$$

$$-2x+1 \qquad -2x \quad +1$$

$$0 = x^2-6x+5$$

$$0=(x-5)(x-1)$$

$$x=5 \qquad x=1$$

Test

$x=5$

$$\sqrt{2\cdot 5-1}+2=5$$

$$\sqrt{9}+2=5$$

$$3+2=5$$

True

$x=1$

$$\sqrt{2\cdot 1-1}+2=1$$

$$\sqrt{1}+2=1$$

$$3=1$$

False

$$\boxed{x=5}$$

(b) $2x^2-x=1$

$$2x^2-x-1=0$$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot 2\cdot(-1)}}{2\cdot 2}=\frac{1\pm\sqrt{1+8}}{4}=\frac{1\pm\sqrt{9}}{4}$$

$$=\frac{1\pm 3}{4} \quad \frac{4}{4}=\boxed{1} \qquad \frac{-2}{4}=\boxed{-\frac{1}{2}}$$

7. (5 points) Let $f(x)=2+3\sqrt{1-x}$ and $h(x)=\frac{2x}{x-3}$

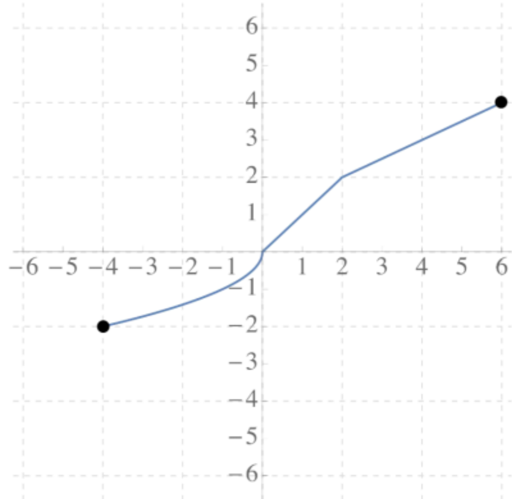
(a) Find $f(1)$

$$f(1)=2+3\sqrt{1-1}=2+3\cdot 0=\boxed{2}$$

(b) Find $h(6)$

$$h(6)=\frac{2\cdot 6}{6-3}=\frac{12}{3}=\boxed{4}$$

8. (10 points) Consider the following function.




(a) Find the domain and range of the graph of the function.

Dom: $[-4, 6]$

Range: $[-2, 4]$

(b) Find $f(2)$ and $f(-4)$.

$f(2) = \boxed{2}$ , $f(-4) = \boxed{-2}$

9. (15 points) Consider the line $6x - 3y + 4 = 0$ and

(a) Find the slope of the given line.

$6x - 3y + 4 = 0$

$-6x \quad -4 \quad -6x - 4$

$-3y = -6x - 4$

$y = \frac{-6}{-3}x + \frac{-4}{-3} = \boxed{2}x + \frac{4}{3}$

$\boxed{\text{slope is } 2}$

(b) Find the equation of the line that is perpendicular to the given line and passes though the point $(4, 1)$. Find the y-intercept of this line.

$m = \frac{-1}{2}$

$y - 1 = \frac{-1}{2}(x - 4)$

$y = \frac{-1}{2}x + \frac{4}{2} + 1$

$\boxed{y = \frac{-1}{2}x + 3}$

$\boxed{\text{y-intercept is } (0, 3)}$

10. (10 points) Consider the circle given by

$$x^2 + y^2 - 4x - 12y - 9 = 0$$

Draw an alien that has:

(a) The number of hands the same as the circle's radius.

(b) The number of legs the same as circle's center y-coordinate.

(c) The number of eyes the same as circle's center x-coordinate.

$$x^2 - 4x + (-2)^2 + y^2 - 12y + (-6)^2 = 9 + (-2)^2 + (-6)^2$$

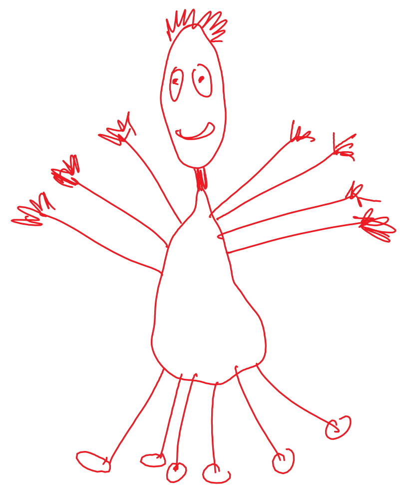
$$(x-2)^2 + (y-6)^2 = 9 + 4 + 36$$

$$(x-2)^2 + (y-6)^2 = 49 \rightarrow \text{center. } (2,6)$$

(a) radius $= \sqrt{49} = \boxed{7}$

(b) y-coordinate: $\boxed{6}$

(c) x-coordinate: $\boxed{2}$



11. (5 points) Simplify

$$\sqrt{40} + 3\sqrt{10}$$

$$\sqrt{40} + 3\sqrt{10} = \sqrt{4} \cdot \sqrt{10} + 3\sqrt{10} = 2\sqrt{10} + 3\sqrt{10}$$

$$= \boxed{5\sqrt{10}}$$

12. (10 points) Simplify

$$\frac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1} = \frac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1} = \frac{\frac{x}{x-2}+1}{\frac{3}{(x-2)(x+2)}+1}$$

$$= \frac{\frac{x}{x-2}+\frac{x-2}{x-2}}{\frac{3}{(x-2)(x+2)}+\frac{(x-2)(x+2)}{(x-2)(x+2)}} = \frac{\frac{x+x-2}{x-2}}{\frac{3+(x-2)(x+2)}{(x-2)(x+2)}}$$

$$= \frac{\frac{2x-2}{x-2}}{\frac{3+x^2-4}{(x-2)(x+2)}} = \frac{2x-2}{\cancel{x-2}} \cdot \frac{\cancel{(x-2)}(x+2)}{3+x^2-4}$$

$$= \frac{(2x-2)(x+2)}{x^2-1} = \frac{2\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+1)} = \boxed{\frac{2(x+2)}{x+1}}$$