MAC 1105, Fall 2017

Exam #1

October 3, 2017

Name Key

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who uses a cell phone during the examination or if one is found within hands reach.
- Calculators are not allowed on this exam.
- The exam consists of two parts. Part I contains four multiple choice questions worth 5 points each. Part II contains 8 open ended questions.

Part I

Choose your answer from five available choices. No partial credit will be given for wrong answers.

1. Simplify $\dfrac{\sqrt{12x^2}}{6x^2}$

   (a) $\dfrac{\sqrt{12}}{6}$

   (b) $\sqrt{3}$

   (c) $\dfrac{2}{1}$

   (d) $\dfrac{\sqrt{3}}{3x}$

   (e) None of the above

$\dfrac{\sqrt{12x^2}}{6x^2} = \dfrac{\sqrt{4}\cdot\sqrt{3}\cdot\sqrt{x^2}}{6x^2} = \dfrac{2\sqrt{3}\cdot x}{6x^2} = \dfrac{\sqrt{3}}{3x}$

2. Divide the following complex numbers and express the result in standard form, $a + bi$.

$\dfrac{2i-3}{2+i}\cdot\dfrac{2-i}{2-i} = \dfrac{4i-2i^2-6+3i}{4-i^2}$

$= \dfrac{4i-2(-1)-6+3i}{4-(-1)} = \dfrac{4i+2-6+3i}{4+1}$

$= \dfrac{7i-4}{5} = \dfrac{-4}{5} + \dfrac{7}{5}i$

   (a) $\dfrac{4-7i}{5}$

   (b) $-\frac{4}{5} + \frac{7}{5}i$

   (c) $-\frac{8}{5} + \frac{7}{5}i$

   (d) $-4 + 7i$

   (e) None of the above

3. Find the solution set for the equation

$$(x-1)^2 = -9$$

   (a) $\{1 + 3i\}$

   (b) $\{1 + 3i, 1 - 3i\}$

   (c) $\{3i + 1, 3i - 1\}$

   (d) The solution set is empty.

   (e) None of the above

$x-1 = \pm\sqrt{-9}$

$x-1 = \pm i\sqrt{9} = \pm i\cdot 3$

$x-1 = \pm 3i$

$x = 1 \pm 3i$

4. Determine the number and type of solutions for the following equation

$$x^2 - 3x - 5 = 0$$

discriminant:
$(-3)^2 - 4 \cdot 1 \cdot (-5) = 9 + 20 = 29 > 0$

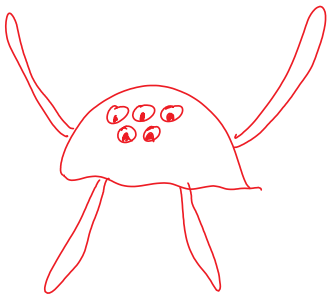
(a) One real solutions.
(b) Two real solution.
(c) Two complex solutions.
(d) Three radical solutions.
(e) None of the above.

Part II

5. (10 points) Consider the points $(1, 6)$ and $(4, 2)$. Draw a squid or octopus that has
   (a) The number of arms the same as the y-coordinate of the midpoint.
   (b) The number of eyes the same as the distance between the points.

(a) midpoint: $\left(\frac{1+4}{2}, \frac{6+2}{2}\right) = \left(\frac{5}{2}, 4\right)$ y-coordinate: $\boxed{4}$

(b) distance $= \sqrt{(1-4)^2 + (6-2)^2} = \sqrt{9+16} = \sqrt{25} = \boxed{5}$



6. (10 points each) Solve for x and include any complex solutions.

(a) $\sqrt{2x-1}+2=x$

$-2 \quad -2$

$\sqrt{2x-1} = x-2 \quad //^2$

$(\sqrt{2x-1})^2 = (x-2)^2$

$2x-1 = x^2-4x+4$

$-2x+1 \quad -2x+1$

$0 = x^2-6x+5$

$0 = (x-5)(x-1)$

$x=5 \quad x=1$

Test

$x=5$

$\sqrt{2\cdot 5-1}+2=5$

$\sqrt{9}+2=5$

$3+2=5$

True

$x=1$

$\sqrt{2\cdot 1-1}+2=1$

$\sqrt{1}+2=1$

$3=1$

False

$\boxed{x=5}$

(b) $2x^2-x=1$

$2x^2-x-1=0$

$x = \frac{-(-1)\pm\sqrt{(-1)^2-4\cdot 2\cdot(-1)}}{2\cdot 2} = \frac{1\pm\sqrt{1+8}}{4} = \frac{1\pm\sqrt{9}}{4}$

$= \frac{1\pm 3}{4}$ < $\frac{4}{4}=\boxed{1}$, $\frac{-2}{4}=\boxed{-\frac{1}{2}}$

7. (5 points) Let $f(x)=2+3\sqrt{1-x}$ and $h(x)=\frac{2x}{x-3}$

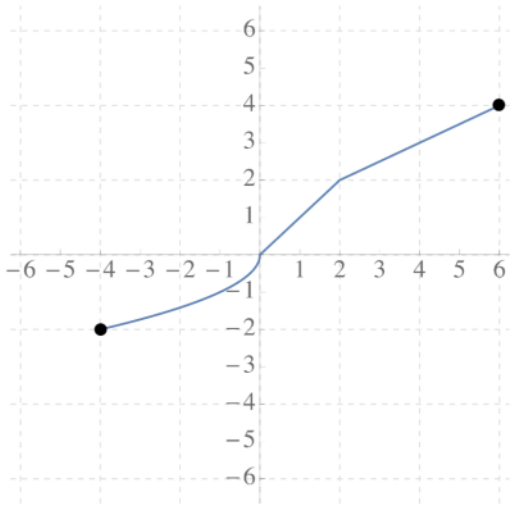
(a) Find $f(0)$

$f(0) = 2+3\sqrt{1-0} = 2+3\cdot 1 = \boxed{5}$

(b) Find $h(4)$

$h(4) = \frac{2\cdot 4}{4-3} = \frac{8}{1} = \boxed{8}$

8. (10 points) Consider the following function.



(a) Find the domain and range of the graph of the function.

Dom: $[-4, 6]$

Range: $[-2, 4]$

(b) Find $f(2)$ and $f(-4)$.

$f(2) = 2$, $f(-4) = -2$

9. (15 points) Consider the line $6x - 3y + 4 = 0$ and

(a) Find the slope of the given line.

$6x - 3y + 4 = 0$

$-6x \quad -4 \quad -6x - 4$

$-3y = -6x - 4$

$y = \frac{-6}{-3}x + \frac{-4}{-3} = 2x + \frac{4}{3}$

slope is 2

(b) Find the equation of the line that is perpendicular to the given line and passes though the point $(4, 1)$. Find the y-intercept of this line.

$m = \frac{-1}{2}$

$y - 1 = \frac{-1}{2}(x - 4)$

$y = \frac{-1}{2}x + \frac{4}{2} + 1$

$y = \frac{-1}{2}x + 3$

y-intercept is $(0, 3)$

10. (10 points) Consider the circle given by

$$x^2 + y^2 - 4x - 12y - 9 = 0$$

Draw an alien that has:

(a) The number of hands the same as the circle's radius.

(b) The number of legs the same as circle's center y-coordinate.

(c) The number of eyes the same as circle's center x-coordinate.

$x^2 - 4x + (-2)^2 + y^2 - 12y + (-6)^2 = 9 + (-2)^2 + (-6)^2$

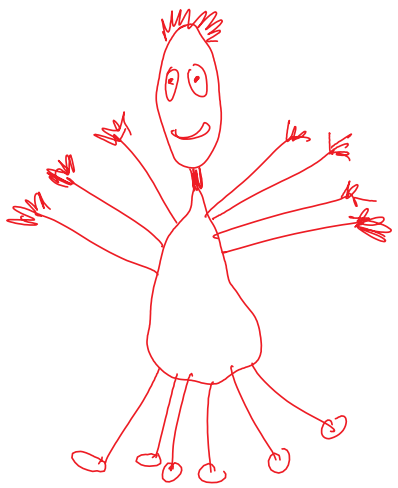
$(x-2)^2 + (y-6)^2 = 9 + 4 + 36$

$(x-2)^2 + (y-6)^2 = 49 \rightarrow$ center. $(2, 6)$

(a) $r = \sqrt{49} = \boxed{7}$

(b) $\boxed{6}$

(c) $= \boxed{2}$



11. (5 points) Simplify

$$\sqrt{40} + 3\sqrt{10}$$

$\sqrt{40} + 3\sqrt{10} = \sqrt{4} \cdot \sqrt{10} + 3\sqrt{10} = 2\sqrt{10} + 3\sqrt{10}$

$= \boxed{5\sqrt{10}}$

12. (10 points) Simplify

$$\frac{\frac{x}{x-2}-1}{\frac{3}{x^2-4}+1}$$

$$\frac{\frac{x}{x-2}-\frac{x-2}{x-2}}{\frac{3}{x^2-4}+\frac{x^2-4}{x^2-4}} = \frac{\frac{x-(x-2)}{x-2}}{\frac{3+x^2-4}{x^2-4}} = \frac{\frac{2}{x-2}}{\frac{x^2-1}{x^2-4}} = \frac{2}{x-2}\cdot\frac{x^2-4}{x^2-1}$$

$$= \frac{2}{x-2}\cdot\frac{(x-2)(x+2)}{x^2-1} = \boxed{\frac{2(x+2)}{x^2-1}}$$