MAC 1105, Fall 2017

Exam #4

December 7, 2017

Name _____

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who uses a cell phone during the examination or if one is found within hands reach.
- Calculators are not allowed on this exam.
- The exam consists of two parts. Part I contains four multiple choice questions worth 5 points each if not stated otherwise. Part II contains six open ended questions worth 9 points each if not stated otherwise.

Honor Code: *On my honor, I have neither received nor given any aid during this examination.*

Signature: _____

Part I

Choose your answer from five available choices. No partial credit will be given for wrong answers.

1. Find $f(g(x))$ if $f(x) = \sqrt{3x-1}$ and $g(x) = x^2 - x$

   (a) $\sqrt{3x^2 - x - 1}$
   (b) $\sqrt{3(x^2 - x) - 1}$
   (c) $\sqrt{3x-1}^2 - \sqrt{3x-1}$
   (d) $\sqrt{3x-1}(x^2 - x)$
   (e) None of the above

$f(x^2-x) = \sqrt{3(x^2-x)-1}$

2. Find the inverse of $f(x) = \frac{2x-1}{3}$.

   (a) $f^{-1}(x) = \frac{3x+1}{2}$
   (b) $f^{-1}(x) = \frac{2x+1}{x}$
   (c) $f^{-1}(x) = \frac{2x-1}{3}$
   (d) $f^{-1}(x) = \frac{3}{2x-1}$
   (e) None of the above

$x = \frac{2y-1}{3}$

$3x = 2y - 1$

$2y = 3x + 1$

$y = \frac{3x+1}{2}$

3. The equation $y = \ln(x)$ is equivalent to which equation?

   (a) $x = \ln(y)$
   (b) $y = b^x$
   (c) $y = e^x$
   (d) $x = e^y$
   (e) None of the above

4. The expression $\ln\left(\frac{x^2}{2^8 y^3}\right)$ can be expanded into

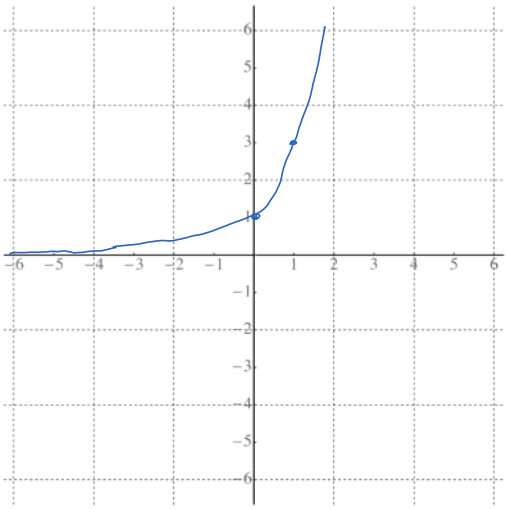
   (a) $2\ln x - 8 \cdot 3\ln 2 + \ln y$
   (b) $\ln x^2 - 8\ln 2 + 3\ln y$
   (c) $2\ln x - 8\ln 2 + 3\ln y$
   (d) $2\ln x - 8\ln 2 - 3\ln y$
   (e) None of the above

Part II

5. Graph $y = -\frac{1}{2} \cdot 3^{x-3}$ using transformations. Start with the graph of a basic function – **plot accurately as least two points** and use them to perform transformations. Do one transformation at a time. Name the transformation and write the equation of the resulting function.

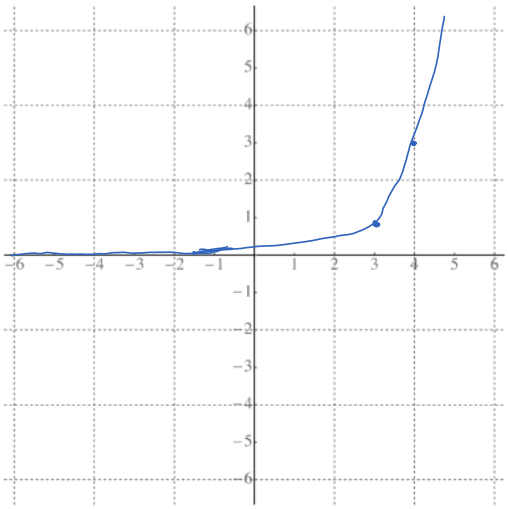
(i) Basic function:

y= 3^x



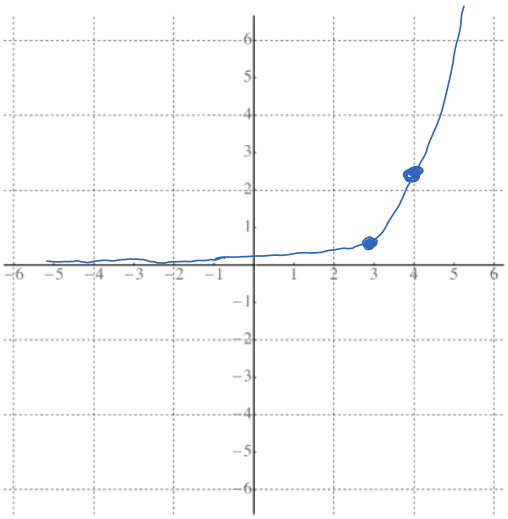
(ii) transformation: hor. shift right by 3

y= 3^{x-3}



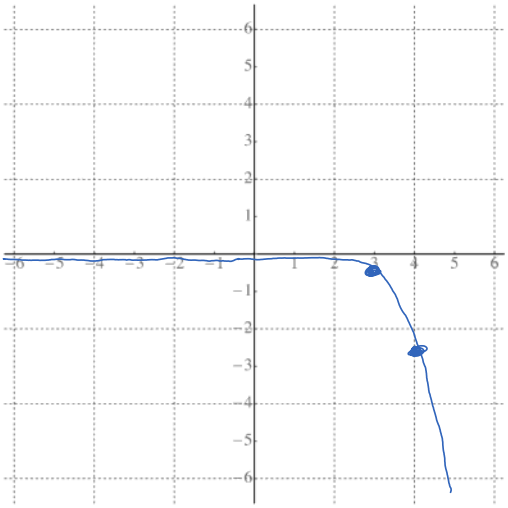
(iii) transformation: vert. shrink by factor of 2

y= $\frac{1}{2} \cdot 3^{x-3}$



(iv) transformation: reflection about the x-axis

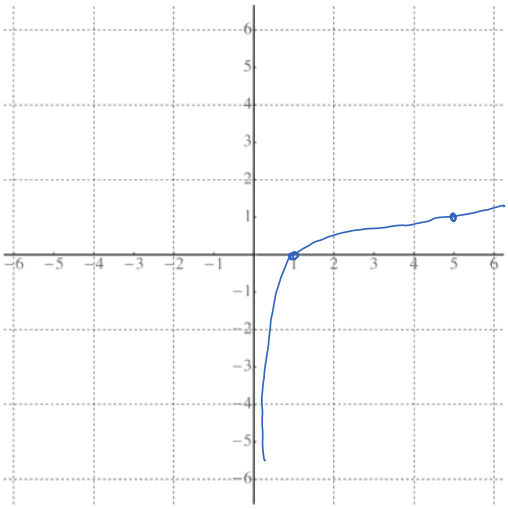
y= $-\frac{1}{2} 3^{x-3}$



6. Graph $y = 4\log_5(\frac{x}{2} + 3)$ using transformations. Start with the graph of a basic function – **plot accurately as least two points** and use them to perform transformations. Do one transformation at a time. Name the transformation and write the equation of the resulting function.

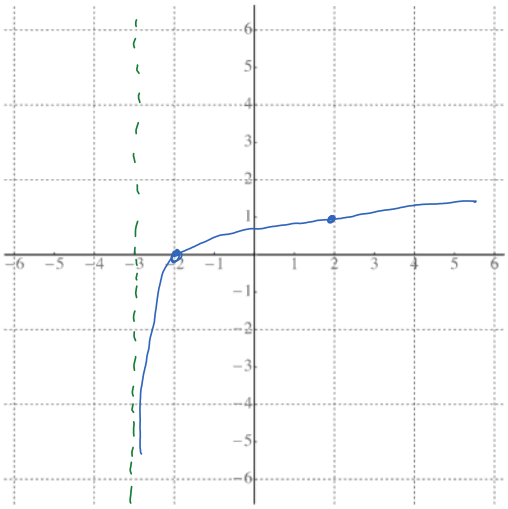
(i) Basic function:

y= $\log_5 x$



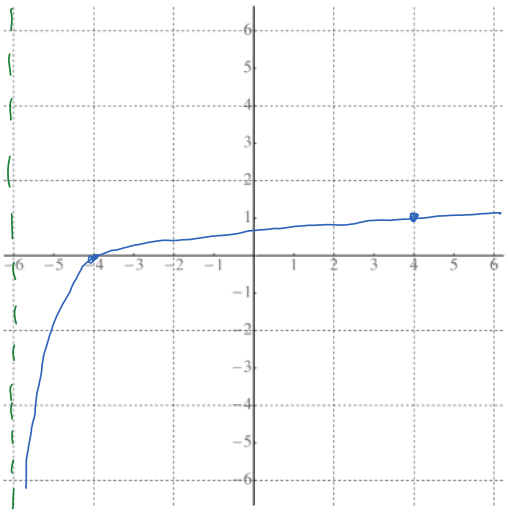
(ii) transformation: hor. shift left by 3

y= $\log_5(x+3)$



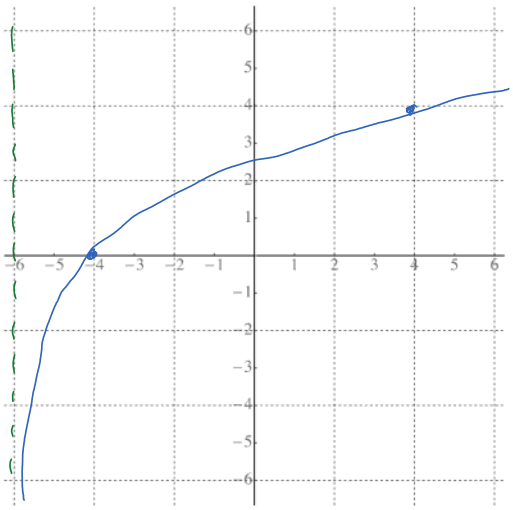
(iii) transformation: hor. stretch

y= $\log_5(\frac{x}{2}+3)$



(iv) transformation: vertical stretch by fact. of 4

y= $4\log_5(\frac{x}{2}+3)$



7. Evaluate the following logarithm

$\log_5(125) = \log_5(5^3) = 3 \cdot \log_5 5$

$= 3 \cdot 1 = \boxed{3}$

8. Rewrite the following expression as one logarithm

$$2\log_3 x + \log_3 y - 4\log_3(x-1) - 3\log_3 z$$

$= \log_3 x^2 + \log_3 y - \log_3 (x-1)^4 - \log_3 z^3$

$= \log_3(x^2 y) - (\log_3 (x-1)^4 + \log_3 z^3)$

$= \log_3(x^2 y) - \log_3[(x-1)^4 z^3] = \boxed{\log_3\left(\frac{x^2 y}{(x-1)^4 z^3}\right)}$

9. Use the properties of logarithms to evaluate

$$\log(25) + \log(4) + \log(10)$$

$= \log(25 \cdot 4 \cdot 10) = \log(100 \cdot 10)$

$= \log 1000 = \log 10^3 = 3\log 10 = \boxed{3}$

10. (9 pts each) Solve the following equation

(a)

$$5^{x-3} = 25$$

$5^{x-3} = 5^2$

$x - 3 = 2$

$\boxed{x = 5}$

(b)

$$2^{2x-1} = \left(\frac{1}{2}\right)^{x+2}$$

$$2^{2x-1} = \left(2^{-1}\right)^{x+2}$$

$$2^{2x-1} = 2^{-x-2} \rightarrow$$

$$2x-1 = -x-2$$

$$3x = -1$$

$$\boxed{x = -1/3}$$

(c)

$$11^{x+1} = 3$$

$$\ln 11^{x+1} = \ln 3$$

$$(x+1)\ln 11 = \ln 3$$

$$x+1 = \frac{\ln 3}{\ln 11}$$

$$\boxed{x = \frac{\ln 3}{\ln 11} - 1} \quad \text{or} \quad \boxed{x = \log_{11}(3) - 1}$$

or $\boxed{x = \dfrac{\ln 3 - \ln 11}{\ln 11}}$

(d)

$$3^{x^2-20} = 9^{4x}$$

$$3^{x^2-20} = \left(3^2\right)^{4x} = 3^{8x}$$

$$x^2 - 20 = 8x$$

$$x^2 - 8x - 20 = 0$$

$$(x-10)(x+2) = 0$$

$$\boxed{x = 10,\ x = -2}$$

(e)

$\log_5(x+23)+\log_5(x-1)=2$

$\log_5((x+23)(x-1)) = 2$

$(x+23)(x-1) = 5^2$

$x^2+22x-23 = 25$

$x^2+22x-48=0$

$(x+24)(x-2)=0$

$x=-24, 2$

check:

$x=-24$

$\log_5(-24+23)+\log_5(-24-1)$

both negative.

$x=2$

$\log_5(2+23)+\log_5(2-1)=2$

$2+0=2$ ✓

Solution: $x=2$