10/05

Thursday, October 5, 2017 4:59 PM

Absolute value

$|-3| = 3$, $|101| = 101$

The distance between a, b is $|a-b|$

Note: $\frac{|a+b|}{2} \stackrel{?}{=}$ midpoint $\boxed{NO}$

- $3, 7 \rightarrow$ distance $|3-7| = |-4| = 4$

$$\frac{|3+7|}{2} = \frac{10}{2} = 5$$

- $-4, 11 \rightarrow$ distance: $|-4-11| = |-15| = 15$

$$\frac{|-4+11|}{2} = \frac{|7|}{2} = 3.5$$

- $-4, 1 \rightarrow$ distance: $|1-(-4)| = |5| = 5$

$$\frac{|-4+1|}{2} = \frac{|-3|}{2} = 1.5$$

Difference quotient

Given a function f, the difference quotient is given by

$$\frac{f(x+h) - f(x)}{}$$

$$\frac{f(x+h)-f(x)}{h}.$$

Find the difference quotient (simplify)

- $f(x)=3x$

$$\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)-3x}{h}=\frac{\cancel{3x}+3h-\cancel{3x}}{h}$$

$$=\frac{3h}{h}=\boxed{3}$$

$$\frac{h+3}{\cancel{h}}\neq\frac{1+3}{1}$$

- $f(x)=x^2-2x$

$$\frac{f(x+h)-f(x)}{h}=\frac{\overbrace{(x+h)^2-2(x+h)}^{f(x+h)}-\overbrace{(x^2-2x)}^{f(x)}}{h}$$

- $f(x)=2x+1$

$y=2x+1$

Find the y-coordinace corresp. to $x=2$. (replace x with 2)

$y=2\cdot2+1=5$

- Find the function value when $x=2$.

$f(2)=2\cdot2+1=5$

$f(x)=x^2+x-2$

$$f(x) = x^2 + x - 2$$

Find $f(1) = (1)^2 + (1) - 2 = 1 + 1 - 2 = \boxed{0}$

$$f(3) = (3)^2 + (3) - 2 = 9 + 3 - 2 = \boxed{10}$$

$$f(-2) = (-2)^2 + (-2) - 2 = 4 - 2 - 2 = \boxed{0}$$

$$f(a) = a^2 + a - 2$$

$$f(0) = 0^2 + 0 - 2$$

$$f(x+h) = (x+h)^2 + (x+h) - 2$$

$$f(x) = x^2 - 2x$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2\cdot(x+h) - (x^2 - 2x)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h - \cancel{x^2} + \cancel{2x}}{h}$$

$$= \frac{2xh + h^2 - 2h}{h} = \frac{\cancel{h}(2x + h - 2)}{\cancel{h}} = \boxed{2x + h - 2}$$

$$f(x) = \sqrt{2x}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

$(a-b)(a+b)$

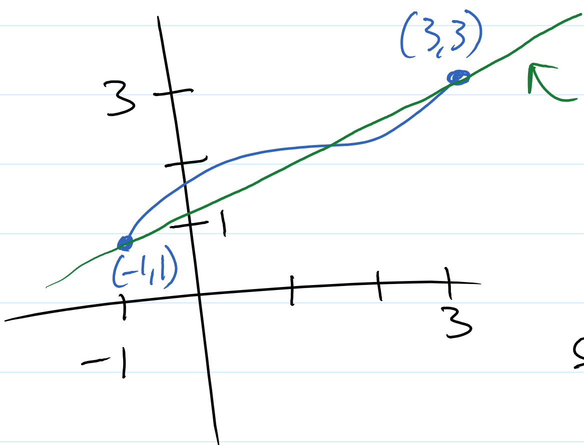
h $\qquad\qquad$ h $\qquad\qquad$ $(a-b)(a+b)$

$$= \frac{\sqrt{2x+2h} - \sqrt{2x}}{h} \cdot \frac{\sqrt{2x+2h} + \sqrt{2x}}{\sqrt{2x+2h} + \sqrt{2x}}$$

$$= \frac{\left(\sqrt{2x+2h}\right)^2 - \left(\sqrt{2x}\right)^2}{h\left(\sqrt{2x+2h} + \sqrt{2x}\right)} = \frac{\cancel{2x}+2h-\cancel{2x}}{h\left(\sqrt{2x+2h} + \sqrt{2x}\right)}$$

$$= \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2x+2h} + \sqrt{2x}\right)} = \boxed{\frac{2}{\sqrt{2x+2h} + \sqrt{2x}}}$$

Average rate of change of a function



slope is the average rate of change of $f(x)$ between $x=-1$, and $x=3$.

slope: $\dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{y_2 - y_1}{x_2 - x_1}$

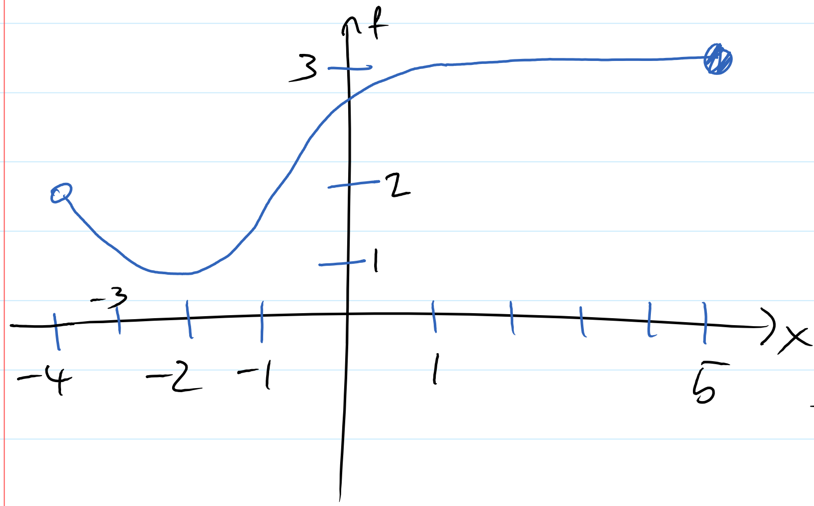
$$\frac{3-1}{3-(-1)} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

The average rate of change of $f(x)$ between x_1 and x_2

$$\frac{f(x_1) - f(x_2)}{\quad} = \frac{f(x_2) - f(x_1)}{\quad}$$

$$\frac{f(x_1)-f(x_2)}{x_1-x_2}=\frac{f(x_2)-f(x_1)}{x_2-x_1}$$

Increasing / decreasing / constant



decreasing on $(-4,-2)$
increasing on $(-2,1)$
constant on $(1,5)$

__Def:__ A function defined on an open interval I is

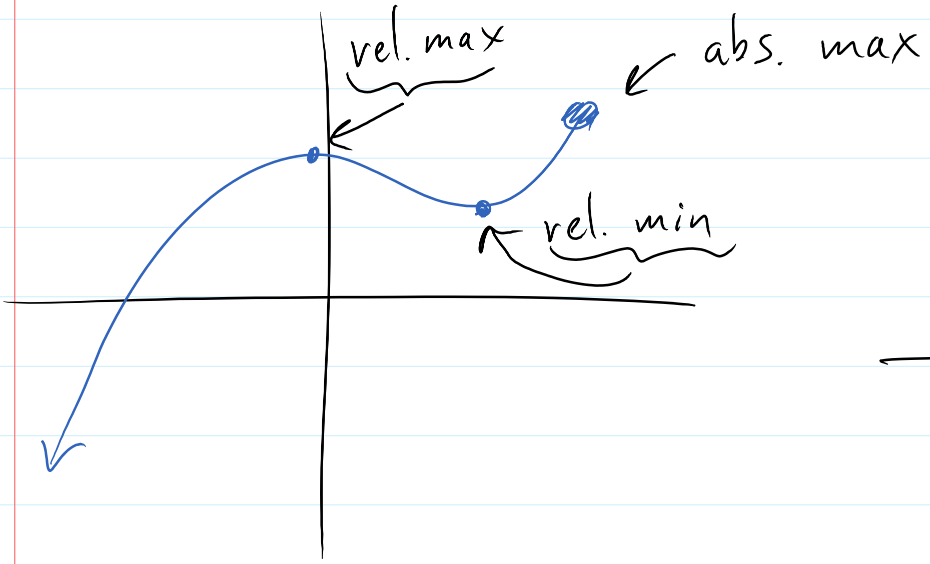
- increasing if $f(x_1) < f(x_2)$ for all $x_1 < x_2$ in I
- decreasing if $f(x_1) > f(x_2)$ for all $x_1 < x_2$ in I
- constant if $f(x_1) = f(x_2)$ for all $x_1 < x_2$ in I.

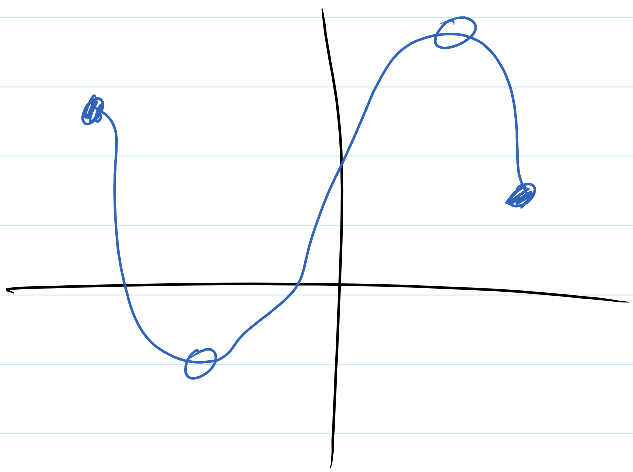
__Def:__ relative maximum or min:

A function f defined on an open interval I, has a rel. $\left\{\begin{matrix} \text{max} \\ \text{min} \end{matrix}\right\}$ at $x=c$ if $\left\{\begin{matrix} f(c) > f(x) \\ f(c) < f(x) \end{matrix}\right\}$ for all x in I.

A function f with the domain D has an

A function f with the domain D has an absolute $\begin{Bmatrix} \text{min} \\ \text{max} \end{Bmatrix}$ at $x=c$ if $\begin{Bmatrix} f(c) < f(x) \\ f(c) > f(x) \end{Bmatrix}$ for all x in D.





no absolute min
no abs. max

Symmetry

The graph of the equation is symmetric with respect to $\begin{Bmatrix} \text{y-axis} \\ \text{x-axis} \\ \text{origin} \end{Bmatrix}$ if substituting

$\begin{Bmatrix} -x \text{ for } x \\ -y \text{ for } y \\ -x \text{ for } x,\ -y \text{ for } y \end{Bmatrix}$ in the equation results in an equivalent equation

Ex: $x = y^2 - 1$

y-axis

$-x = y^2 - 1$ $\boxed{NO}$

origin

$-x = (-y)^2 - 1$

$-x = y^2 - 1$ $\boxed{NO}$

x-axis

$x = (-y)^2 - 1$

$x = y^2 - 1$ $\boxed{yes}$

Ex: $y = x^3$

y-axis

$y = (-x)^3$

$y = -x^3$ $\boxed{NO}$

origin

$-y = (-x)^3$

$-y = -x^3 \quad (\cdot -1)$

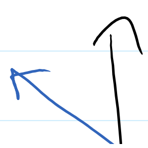
$y = x^3$ $\boxed{yes}$

x-axis

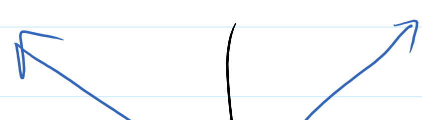
$-y = x^3 \quad / \cdot (-1)$

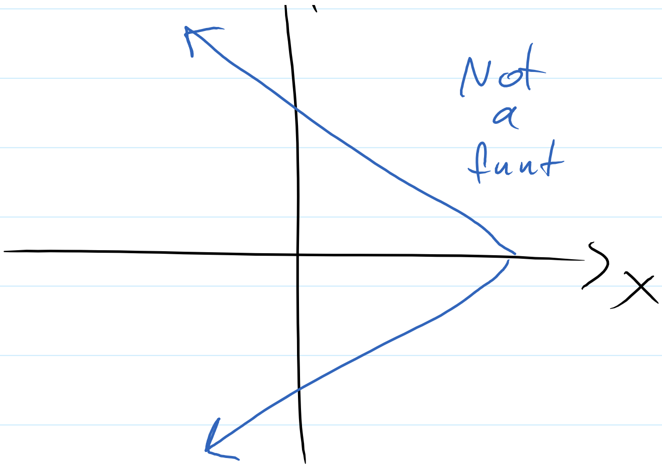
$y = -x^3$ $\boxed{No}$

A function is
- odd if it is sym. with respect to the origin $(f(-x) = -f(x))$
- even if it is sym. with resp to the y-axis $(f(-x) = f(x))$



Not

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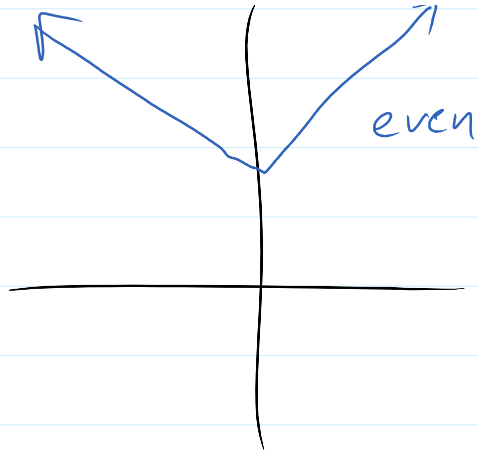
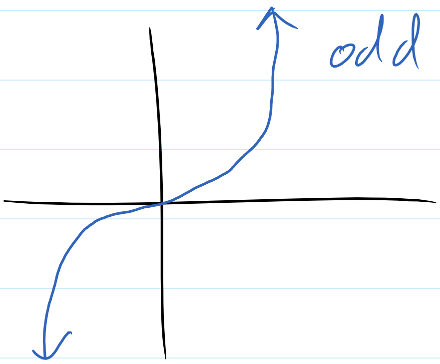
x


even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