

$$\underline{\text{Ex:}} \quad f(x) = 2x - 1 \quad \text{Dom } f: (-\infty, \infty)$$

$$g(x) = x^2 + x - 3 \quad \text{Dom } g: (-\infty, \infty)$$

$$(f+g)(x) = f(x) + g(x) = 2x - 1 + (x^2 + x - 3)$$

$$= x^2 + 3x - 4$$

$$\text{Dom: } (-\infty, \infty)$$

$$(f-g)(x) = 2x - 1 - (x^2 + x - 3)$$

$$= 2x - 1 - x^2 - x + 3 = -x^2 + x + 2$$

$$\text{Dom: } (-\infty, \infty)$$

$$(f \cdot g)(x) = (2x - 1) \cdot (x^2 + x - 3)$$

$$= 2x^3 + 2x^2 - 6x - x^2 - x + 3 = 2x^3 + x^2 - 7x + 3$$

$$\text{Dom: } (-\infty, \infty)$$

$$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{x^2 + x - 3}{2x - 1}$$

$$\text{Dom: } 2x - 1 \neq 0$$

$$2x \neq 1$$

$$x \neq \frac{1}{2}$$

$$\boxed{(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)}$$

Section 3.1 vertical stretch

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Plot: $y = 3(x-1)^2 + 2$

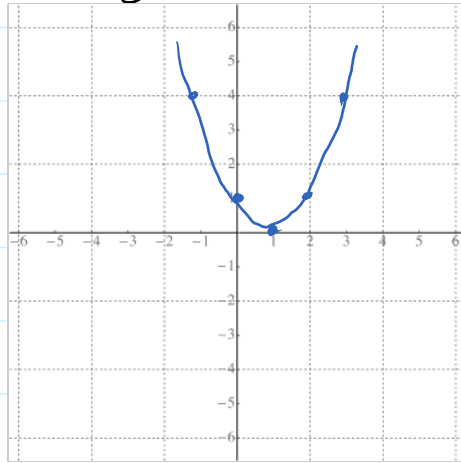
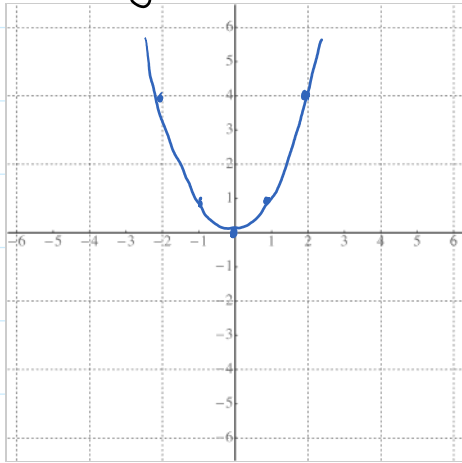
Annotations:
- A green arrow points from "vertical stretch" to the coefficient 3.
- A green arrow points from "right" to the term $(x-1)$.
- A green arrow points from "up" to the constant term +2.

base: x^2

hor. shift to right

$$y = (x-1)^2$$

$$y = x^2$$

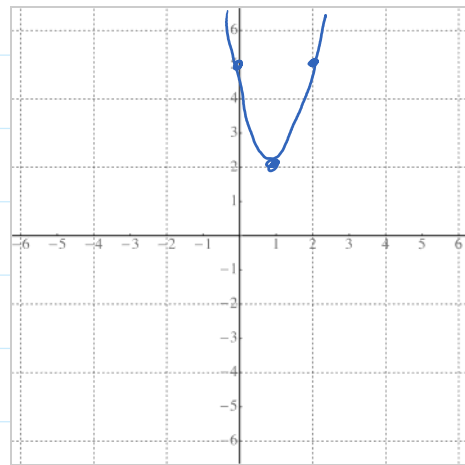
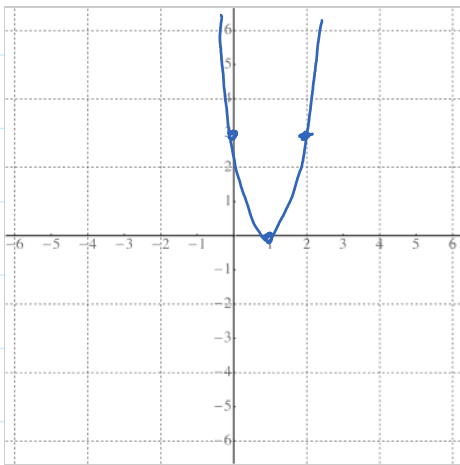


vertical stretch by
fact. of 3

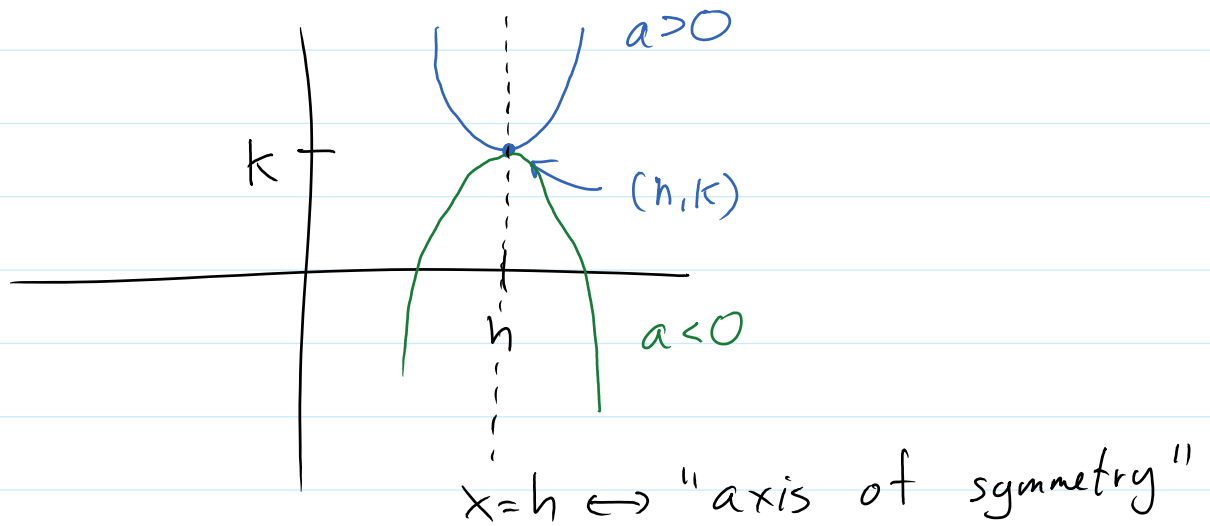
$$y = 3(x-1)^2$$

shift up by 2

$$y = 3(x-1)^2 + 2$$



A parabola $y = a(x-h)^2 + k$ has a vertex at (h, k) :



The general form of a quadratic function (parabola)

$$f(x) = ax^2 + bx + c,$$

the standard form of parabola is

$$a(x-h)^2 + k.$$

Ex: Find the general form of and graph the parabola.

$$f(x) = -2(x-3)^2 + 8$$

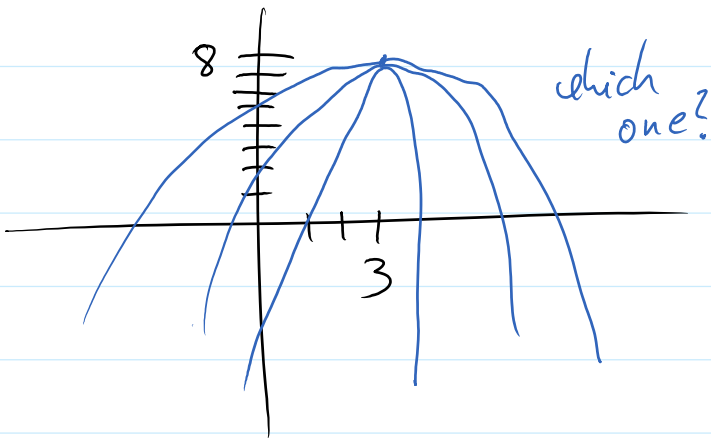
$$f(x) = -2(x-3)^2 + 8 \quad \text{foil } (a-b)^2 = a^2 - 2ab + b^2$$

$$= -2(x^2 - 2 \cdot x \cdot 3 + 9) + 8$$

$$= -2x^2 + 12x - 18 + 8$$

$$\begin{aligned}
 &= -2x^2 + 12x - 18 + 8 \\
 &= -2x^2 + 12x - 10
 \end{aligned}$$

\swarrow
 vertex: $(3, 8)$
 open down



y-int: $(0, f(0))$

$$f(0) = -2 \cdot 0 + 12 \cdot 0 - 10 = -10$$

$(0, -10)$

x-int: $(?, 0)$

need to solve } $f(x) = 0$

$$-2(x-3)^2 + 8 = 0$$

$$\frac{-2(x-3)^2}{-2} = \frac{-8}{-2}$$

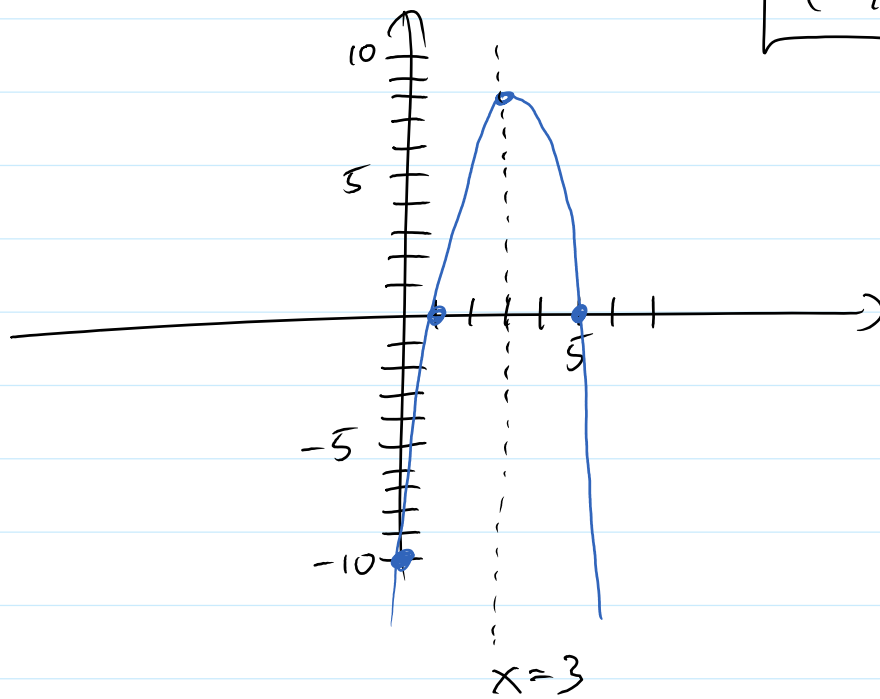
$$(x-3)^2 = 4$$

$$x-3 = \pm\sqrt{4}$$

$$x = 3 \pm 2 \begin{cases} x = 5 \\ x = 1 \end{cases}$$

$(5, 0), (1, 0)$

\updownarrow :



Plot: $f(x) = (x+3)^2 + 1$

vertex: $(-3, 1)$

axis of symmetry: $x = -3$

open: up

y-int: $f(0) = (0+3)^2 + 1 = 10$
 $(0, 10)$

x-int:

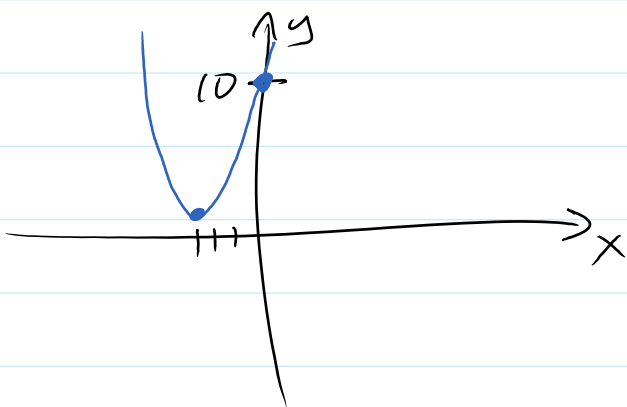
$$(x+3)^2 + 1 = 0$$

$$(x+3)^2 = -1$$

$$x+3 = \pm \sqrt{-1}$$

no real solutions

no x-int



Graph : $f(x) = -x^2 - 2x + 1$

Find the vertex:

by formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

x-coordianance: $\boxed{\frac{-b}{2a}}$

$$x = \frac{-(-2)}{2 \cdot (-1)} = \frac{2}{-2} = -1$$

y-coord: evaluate $f(x)$
at $x = \frac{-b}{2a}$

$$\begin{aligned} f(-1) &= -(-1)^2 - 2(-1) + 1 \\ &= -1 + 2 + 1 \\ &= 2 \end{aligned}$$

$$\boxed{\text{vertex: } (-1, 2)}$$

complete the square:

$$-(x^2 + 2x - 1)$$

add & sub: $\left(\frac{+2}{2}\right)^2 = 1^2 = 1$

$$-(x^2 + 2x + 1 - 1 - 1)$$

$$\begin{aligned} &-(x^2 + 2x + 1) + 1 + 1 \\ &-(x+1)^2 + 2 \end{aligned}$$

$$\boxed{\text{vertex: } (-1, 2)}$$

Ex: Plot:

$$f(x) = 3x^2 - 2x - 4$$

vertex: $x = \frac{-(-2)}{2 \cdot 3} = \frac{2}{6} = \frac{1}{3}$

$$y = f\left(\frac{1}{3}\right) = 3 \cdot \left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right) - 4 = \frac{3}{1} \cdot \frac{1}{9} - \frac{2}{3} - 4$$

$$= \frac{1}{3} - \frac{2}{3} - \frac{12}{3} = \frac{-13}{3} \approx -4.33$$

$$\boxed{\left(\frac{1}{3}, \frac{-13}{3}\right)}$$

y-int: $f(0) = -4$ $(0, -4)$

x-int: $\frac{2 \pm \sqrt{4 + 4 \cdot 3 \cdot 4}}{2 \cdot 3} = \frac{2 \pm \sqrt{4 + 3 \cdot 16}}{6}$

$$= \frac{2 \pm \sqrt{52}}{6} = \frac{2 \pm \sqrt{4 \cdot 13}}{6} = \frac{2 \pm 2\sqrt{13}}{6}$$

$$= \frac{2(1 \pm \sqrt{13})}{6} = \frac{1 \pm \sqrt{13}}{3}$$

