Odd / even function:

$f(-x) = f(x) \rightarrow$ even / sym. about the y-axis

$f(-x) = -f(x) \rightarrow$ odd / sym. about the origin

• $f(x) = x^3 + 3x$

$$f(-x) = (-x)^3 + 3(-x) = -x^3 - 3x = -\overbrace{(x^3+3x)}^{f(x)}$$

$\boxed{\text{ODD}}$

• $f(x) = x^4 + 2x^2 - 1$

$$f(-x) = (-x)^4 + 2(-x)^2 - 1 = \overbrace{x^4 + 2x^2 - 1}^{f(x)}$$

$\boxed{\text{Even}}$

• $g(x) = x^5 - x^2 - 2$

$$g(-x) = (-x)^5 - (-x)^2 - 2 = -x^5 - x^2 - 2 = -(x^5 + x^2 + 2)$$

$\boxed{\text{not sym.}}$

• $g(x) = \dfrac{x^3 - x}{2x^2}$

$$g(-x) = \frac{(-x)^3 - (-x)}{2(-x)^2} = \frac{-x^3 + x}{2x^2} = \frac{-x^3 + x}{-(-2x^2)} = -\frac{-x^3 + x}{-2x^2}$$

$$= \frac{-(x^3-x)}{2x^2} = -\underbrace{\frac{x^3-x}{2x^2}}_{f(x)}$$

$\boxed{\text{ODD}}$

Find the difference quotient of

- $f(x) = 2x^2 - 5$

$$\frac{f(x+h)-f(x)}{h}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2-5-(2x^2-5)}{h}$$

$$= \frac{2(x^2+2xh+h^2)\cancel{-5}-2x^2\cancel{+5}}{h}$$

$$= \frac{\cancel{2x^2}+4xh+2h^2\cancel{-2x^2}}{h} = \frac{4xh+2h^2}{h} = \frac{\cancel{h}(4x+2h)}{\cancel{h}}$$

$$= \boxed{4x+2h}$$

Find the average rate of change of $f(x) = \frac{2}{x}$ from $x_1 = 1$ to $x_2 = 4$

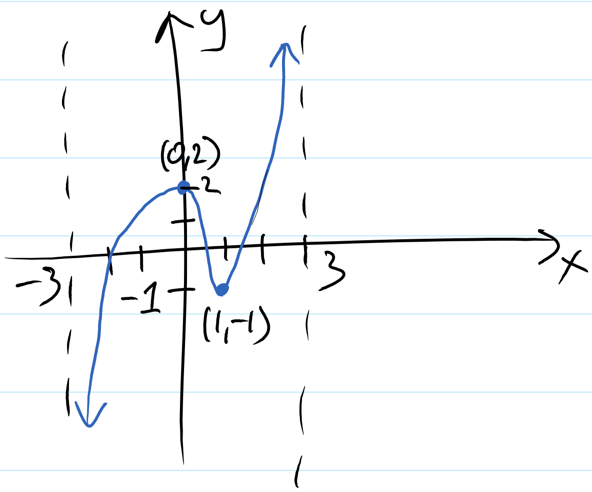
$(x_1, f(x_1))$, $(x_2, f(x_2))$

$(1, 2)$, $(4, \frac{1}{2})$

$(1, 2)$, $(4, \frac{1}{2})$

ave. rate of change: $\dfrac{f(x_1) - f(x_2)}{x_1 - x_2} = \dfrac{2 - \frac{1}{2}}{1 - 4}$

$= \dfrac{1.5}{-3} = \dfrac{3/2}{-3/1} = \dfrac{\cancel{3}}{2} \cdot \dfrac{-1}{\cancel{3}} = \boxed{\dfrac{-1}{2}}$



Dom: $(-3, 3)$

Range: $(-\infty, \infty)$

zeros of $f(x)$: $(-2, 0), (\frac{1}{2}, 0), (\frac{3}{2}, 0)$

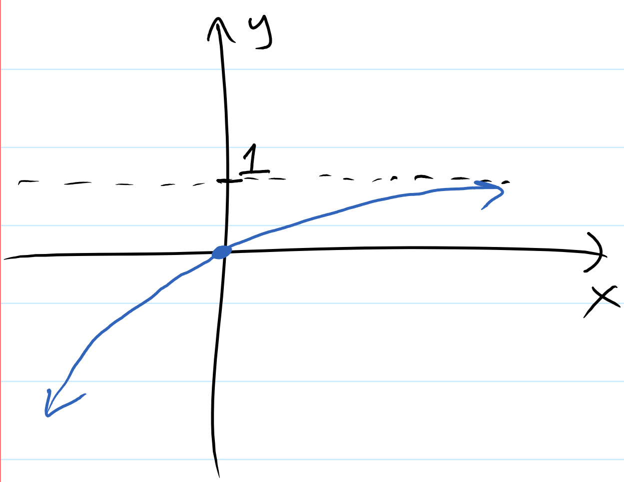
(cross the x-axis)

int. of inc.: $(-3, 0) \cup (1, 3)$

int. of dec: $(0, 1)$

rel. min: $(1, -1)$ or $x = 1$

rel. max: $(0, 2)$ or $x = 0$



Dom: $(-\infty, \infty)$

Range: $(-\infty, 1)$

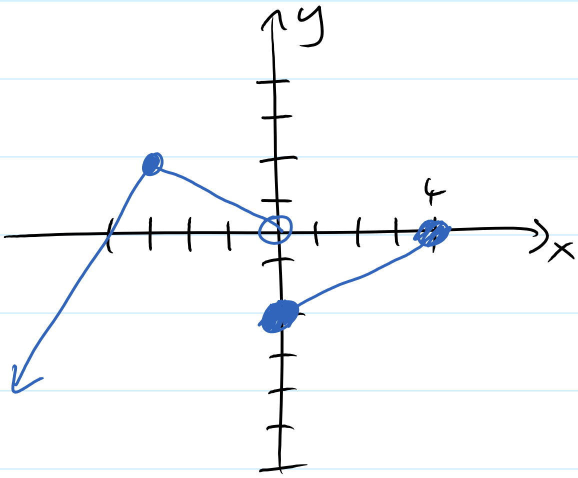
zeros: $(0, 0)$

inc: $(-\infty, \infty)$

dec: None

rel. min: none

rel. max: none



Dom: $(-\infty, 4]$

Range: $(-\infty, 2]$

zeros: $(-4,0), (4,0)$

inc: $(-\infty,-3) \cup (0,4)$

dec: $(-3,0)$

rel. min: $(0,-2)$

rel. max: $(-3,2)$

$$F(x) = \begin{cases} x^2, \text{ if } & x \le -2 \\ 0, \text{ if } & -2 < x \le 0 \\ -x, \text{ if } & x > 0 \end{cases}$$

$F(-5) = (-5)^2 = 25$

$F(-2) = (-2)^2 = 4$

$F(1.9) = -1.9$

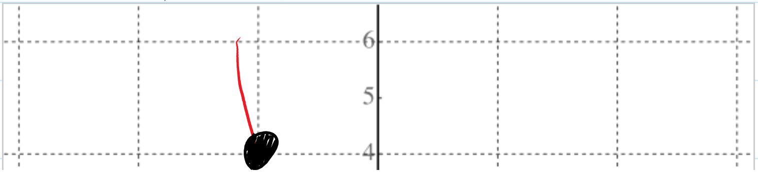
$F(-1.9) = 0$

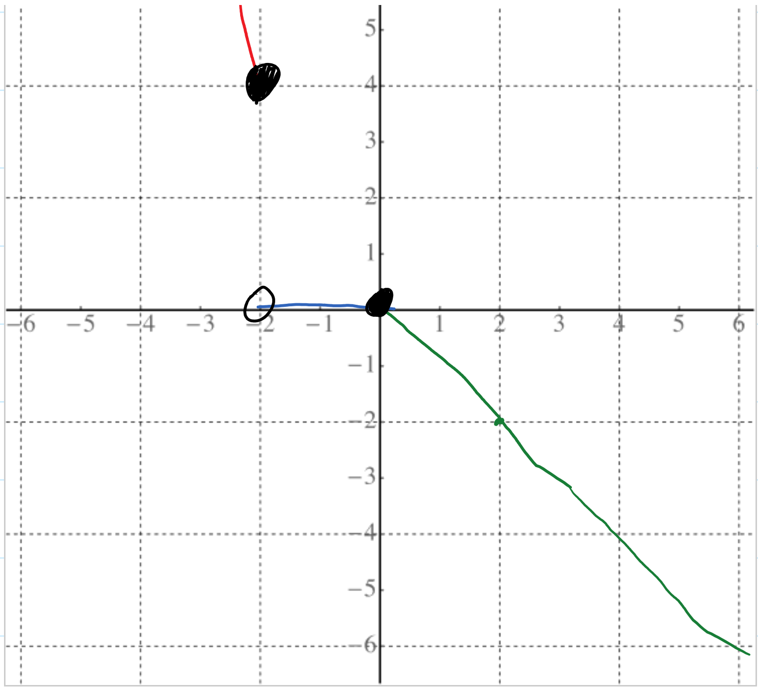
$F(-1) = 0$

$F(1) = -1$

$F(3) = -3$

$$F(x) = \begin{cases} x^2, \text{ if } & x \le -2 \\ 0, \text{ if } & -2 < x \le 0 \\ -x, \text{ if } & x > 0 \end{cases}$$



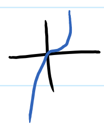


Plot using transformations:

$$f(x) = (2x)^3 - 3$$

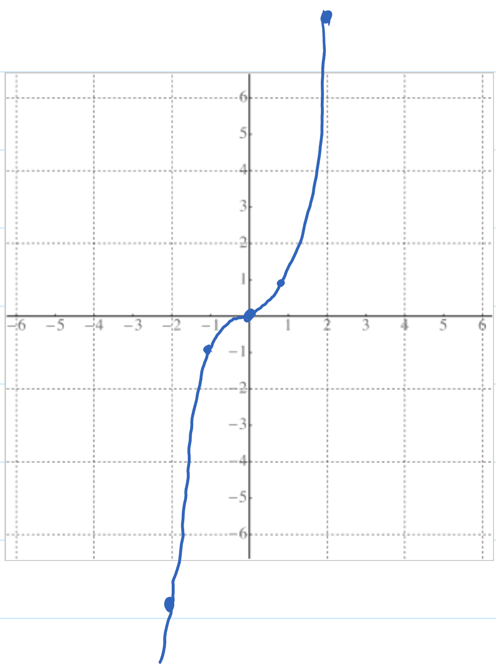
vertical shift down by 3

horizontal shink

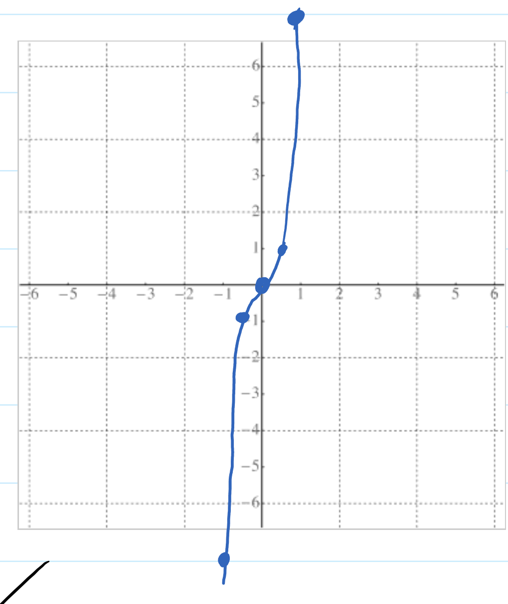
basic func: x^3 

$y = (2x)^3$

hor. shink



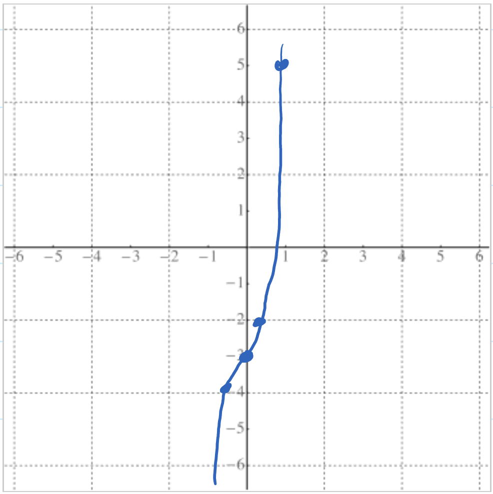
$\rightarrow$



$y = (2x)^3 - 3$

$y=(2x)^3-3$

vertical shift down 3.



$(0,0) \to (0,-3)$

$(0.5,1) \to (0.5,-2)$

$(-0.5,-1) \to (-0.5,-4)$

$(1,8) \to (1,5)$

The point $(6,8)$ is on the graph of $f(x)$. Find out where is the point going to move on the graph of $-\frac{1}{2}f(3x-2)+1$

- reflection about the x-axis
- vertical shrink by 2

hor. shrink by factor of 3.

hor. shift right 2.

vertical shift up 1

$$\underset{f(x)}{(6,8)} \longrightarrow \underset{f(x-2)}{(8,8)} \longrightarrow \underset{f(3x-2)}{\left(\tfrac{8}{3},8\right)} \longrightarrow \underset{\frac{1}{2}f(3x-2)}{\left(\tfrac{8}{3},4\right)}$$

$$\longrightarrow \underset{-\frac{1}{2}f(3x-2)}{\left(\tfrac{8}{3},-4\right)} \longrightarrow \underset{-\frac{1}{2}f(3x-2)+1}{\boxed{\left(\tfrac{8}{3},-3\right)}}$$