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Thursday, October 26, 2017 5:10 PM

Ex: Among all pairs of numbers whose difference is 10, find a pair whose product is as small as possible.

numbers: x, $x+10$

Product: $x(x+10) = x^2 + 10x$

find the minimum of this function.

x^2+10x represents a parabola open up so the vertex is its min.

x-coor: $\frac{-10}{2\cdot 1} = \frac{-10}{2} = -5$

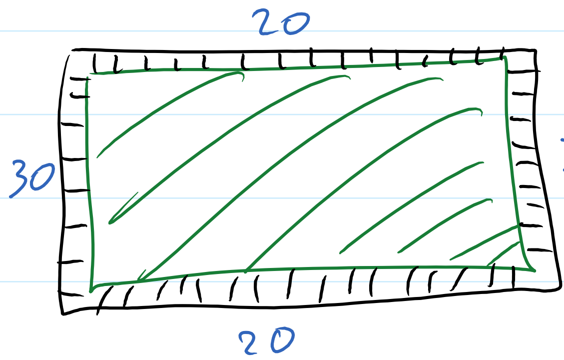
numbers: -5, $-5+10$

$\boxed{-5,\ 5}$

Ex: We have 100 yards of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximizes the enclosed area. What is the maximum area?

area?

$A=600$



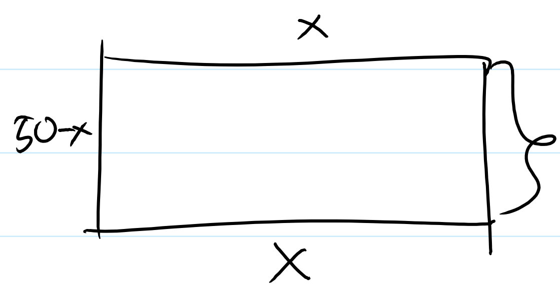
$30 = \frac{100-2\cdot 20}{2}$

$A = 10\cdot 40 = 400$

$10 = \frac{100-2\cdot 40}{2}$

$A = x(50-x)$

$= 50x - x^2$



$\frac{100-2x}{2} = 50-x$

Find the maximum of this function

$A = -x^2 + 50x$

x-coor: $\frac{-50}{2\cdot(-1)} = \frac{-50}{-2} = 25$

The length of the field is $\boxed{25}$ yards,
the width is $50-25=\boxed{25.}$
The maximum area: $25\cdot 25 = \boxed{625}$

$35^2 = 1225$

$75^2 = \underbrace{56}\underbrace{25}$

$25^2 = \underbrace{6}_{2\cdot 3}25$

$45^2 = \underbrace{20}_{4\cdot 5}25$

Ex: Find a parabola that has similar shape as $3x^2$ or $-3x^2$ and maximum $= 4$ at $x = -1$.

$y = a(x-h)^2 + k$

$\boxed{a = -3}$

$(h, k) = (-1, 4)$

$$y = -3(x-(-1))^2 + 4$$

$$\boxed{y = -3(x+1)^2 + 4}$$

Find a quad. func. that has a vertex at $(2, -8)$ and $\underbrace{\text{y-int at } y=6}_{(0,6)}$.

$$y = a(x-2)^2 - 8$$

$(0, 6)$

To find a,
$x=0,\ y=6$

$$6 = a(0-2)^2 - 8$$

$$6 = a \cdot 4 - 8$$

$$+8 \qquad\qquad +8$$

$$14 = 4a$$

$$a = \frac{14}{4} = \frac{7}{2}$$

$$\boxed{y = \frac{7}{2}(x-2)^2 - 8}$$

The price p and quantity x of a product obey the demand equation

$$p = -\frac{1}{8}x + 100$$

a) Find the revenue. $(R = p \cdot x)$

$$R(x) = \left(-\frac{1}{8}x + 100\right) \cdot x = \boxed{-\frac{1}{8}x^2 + 100x}$$

b) What quantity maximizes revenue?

$$x = \frac{-100}{2 \cdot \left(-\frac{1}{8}\right)} = \frac{-100}{-1/4} = \frac{100}{1} \cdot \frac{4}{1} = \boxed{400}$$

A company has fixed cost of \$800, and variable cost of \$2.

a) Find the total cost function.

$$C(x) = (\text{fixed cost}) + x(\text{variable cost})$$
$$= \boxed{800 + 2x}$$

b) The revenue is

$$R(x) = -0.002x^2 + 2.2x - 500,$$

Find the profit function $P(x)$ and its maximum.

$$P(x) = R(x) - C(x)$$
$$= -0.002x^2 + 2.2x - 500 - (800 + 2x)$$
$$= -0.002x^2 + 2.2x - 500 - 800 - 2x$$
$$= \boxed{-0.002x^2 + 0.2x - 1300}$$

$$x = \frac{-0.2}{2(-0.002)} = \frac{-0.2}{-0.004} = \boxed{50}$$